1) Max value of $z = 2x + y$ = objective function

Subject to $x + 2y \leq 10$
$2x + y \leq 6$
$x - y \leq 2$
$x - 2y \leq 1$
} = constraints

$x, y \geq 0$ = non-negativity

Considering each of the functions

i) $x + 2y \leq 10$

When $x = 0$, $y = 5$ } $(0, 5)$
" $y = 0$, $x = 10$ } $(10, 0)$

ii) $x + y \leq 6$

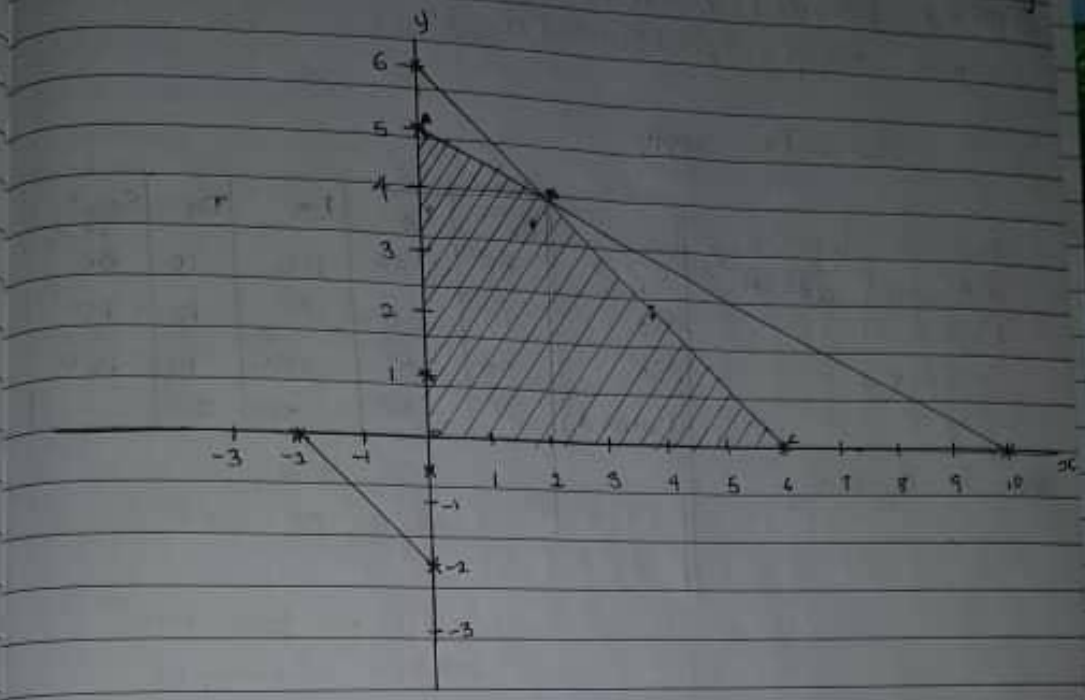
When $x = 0$, $y = 6$ } $(0, 6)$
" $y = 0$, $x = 6$ } $(6, 0)$

) $x - y \leq 2$

When $x = 0$, $y = -2$ } $(0, -2)$
" $y = 0$, $x = -2$ } $(-2, 0)$

) $x - 2y \leq 1$

When $x = 0$, $y = -0.5$ } $(0, -0.5)$
" $y = 0$, $x = 1$ } $(1, 0)$



optimal values:

A (0, 5)

B (2, 3.9)

C (6, 0)

substitute the values into the objective functions

$A = 2(0) + 5$

$= 5$

$B = 2(2) + 3.9$

$= 7.9$

$C = 2(6) + 0$

$= 12$

The maximum point is C

i) The regret method

2)	D_1	D_2	D_3	D_4	Supply	RP1	RP2	RP3	RP4
S_1	20 (10)	30 (50)	110	70	~~70~~ ~~60~~	10	10	50	—
S_2	10	0	60	10 (10)	~~10~~	10	—	—	—
S_3	50 (60)	80	150 (30)	90 (110)	~~100~~ ~~40~~ 10	30	30	40	40
Demand	~~70~~ ~~60~~	~~50~~	30	~~10~~ ~~20~~	= 170 / 170				
cp1	10	30	50	60					
cp2	30	50	40	20					
cp3	30	—	40	20					
~~cp4~~	~~50~~								

$$C = 10(20) + 50(30) + 10(10) + 60(50) + 30(150) + 10(90)$$
$$C = 200 + 1500 + 100 + 3000 + 4500 + 900$$
$$C = 10200$$

ii) Least cost method

	D_1	D_2	D_3	D_4	Supply
S_1	20 (60)	30	110	70	~~60~~
S_2	10 (10)	0 (10)	60 (30)	10 (20)	~~10~~ ~~40~~ ~~30~~
S_3	50	80 (40)	150	90	~~100~~ ~~50~~
Demand	~~70~~ ~~10~~	~~40~~ ~~50~~	~~30~~	~~20~~	

occupied cells = 6

$m+n-1$

m = No of rows = 4

n = No of columns = 3

$4+3-1=6$ This means there is no degeneracy

Total cost

$S_1 \rightarrow D_1 = 20 \times 60 = 1200$

$S_2 \rightarrow D_2 = 0 \times 10 = 0$

$S_3 \rightarrow D_1 = 50 \times 10 = 500$

$S_3 \rightarrow D_2 = 80 \times 40 = 3200$

$S_3 \rightarrow D_3 = 150 \times 30 = 4500$

$S_3 \rightarrow D_4 = 90 \times 20 = 1800$

11200

	D_1	D_2	D_3	D_4	Supply
S_1	20	30	110	70	~~60~~
S_2	10	0	60	10	~~10~~
S_3	50	80	150	90	~~100~~ 70
Demand	~~70~~ 10	~~50~~ 40	~~30~~	20	

b) North-west corner method

	D_1	D_2	D_3	D_4	Supply
S_1	20 ~~60~~	30	110	70	~~60~~
S_2	10 ~~110~~	0	60	10	~~10~~
S_3	50	80 ~~50~~	150 ~~30~~	90 ~~20~~	~~100~~ ~~50~~ ~~20~~
Demand	~~70~~ ~~10~~	~~50~~	~~30~~	~~20~~	

Occupied cells = 5

$m+n-1$

m = No of rows = 4

n = No of colums = 3

$4+3-1 = 6$

since occupied cells $\neq m+n-1$ this means there is degeneracy.

Question 5

I

Bin	1	2	3	4	5	6	7	8	9
1	–	4	–	6	7	–	3	–	5
2	4	–	5	2	–	3	1	–	–
3	–	5	–	7	–	2	2	4	–
4	6	2	7	–	4	1	–	3	–
5	–	–	–	4	–	1	–	–	–
6	–	3	2	1	1	–	2	2	4
7	3	1	2	–	–	2	–	5	2
8	–	–	4	3	–	2	5	–	6
9	5	–	–	–	–	4	2	6	–

Bin	1	2	3	4	5	6	7	8	9
1	–	3	–	5	6	–	2	–	3
2	1	–	3	1	–	2	0	–	–
3	–	4	–	6	–	1	1	2	–
4	3	1	5	–	3	0	–	1	–
5	–	–	–	3	–	0	–	–	–
6	–	2	0	0	0	–	1	0	2
7	0	0	0	–	–	1	–	3	0
8	–	–	2	2	–	1	4	–	4
9	2	–	–	–	–	3	1	4	–

$A_2 \to A_1 \to A_1 \Rightarrow$ Co

Bin	1	2	3	4	5	6	7	8	9
1	–	1	–	3	4	–	0	–	1
2	1	–	3	1	–	2	0	–	–
3	–	3	–	5	–	0	0	1	–
4	3	1	5	–	3	0	–	1	–
5	–	–	–	3	–	0	–	–	–
6	–	2	0	0	0	–	1	0	2
7	0	0	0	–	–	1	–	3	0
8	–	–	1	1	–	0	3	–	3
9	1	–	–	–	–	2	0	3	–

Bin	1	2	3	4	5	6	7	8	9
1	–	0	–	2	3	–	0	–	0
2	0	–	2	0	–	2	0	–	–
3	–	2	–	4	–	0	0	0	–
4	2	0	4	–	2	0	–	0	–
5	–	–	–	2	–	0	–	–	–
6	–	2	0	0	0	–	1	0	2
7	0	0	0	–	–	1	–	3	0
8	–	–	0	0	–	0	3	–	2
9	0	–	–	–	–	2	0	2	–

9 – 7 = 8
8 – 4 = 3
7 – 3 = 2
6 – 5 = 2
6 – 5 = 1
5 – 6 = 1
4 – 2 = 2
3 – 8 = 4

2 – 1 = 4
1 – 9 = 5
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Question 3

a $R_1 \rightarrow R_1 \Rightarrow P_{11} = 0.7$

$R_1 \rightarrow R_2 \Rightarrow P_{12} = 0.3$

$R_2 \rightarrow R_1 \Rightarrow P_{21} = 0.2$

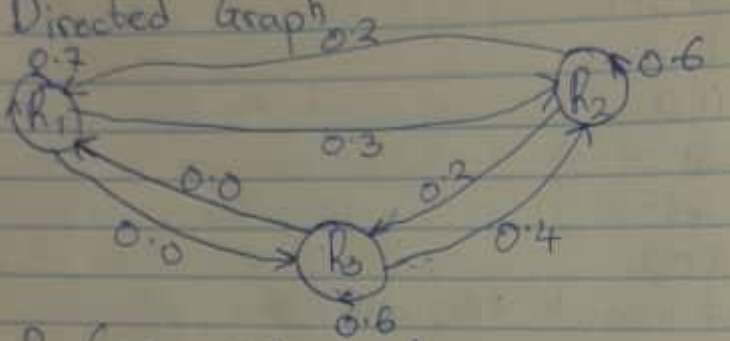
$R_2 \rightarrow R_2 \Rightarrow P_{22} = 0.6$

$R_2 \rightarrow R_3 \Rightarrow P_{23} = 0.2$

$R_3 \rightarrow R_2 \Rightarrow P_{32} = 0.4$

$R_3 \rightarrow R_3 \Rightarrow P_{33} = 0.6$

Directed Graph



$$P = \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$$

b $P^{(1)} = P^{(0)} \cdot P$

$P^{(1)} = (0.3, 0.6, 0.1) \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$

$= (0.33, 0.49, 0.18)$

$P^{(2)} = P^{(1)} \cdot P$

$P^{(2)} = (0.33, 0.49, 0.18) \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$

$= (0.329, 0.465, 0.206)$

c $P = \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$

Sol

$P^{(s)} = (x, y, z)$

$P^{(s)} = P^{(s)} \cdot P$

$(x, y, z) = (x, y, z) \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$

$(x, y, z) = (0.7x + 0.2y + 0z,\ 0.3x + 0.6y + 0.4z,\ 0.0x + 0.2y + 0.6z)$

$x = 0.7x + 0.2y + 0z$ —— ①

$y = 0.3x + 0.6y + 0.4z$ —— ②

$z = 0.0x + 0.2y + 0.6z$ —— ③

$\Sigma P = 1$

because probability is meant to be equal to 1

$P^{(s)} = (x, y, z)$

$x + y + z = 1$ —— ④

From ①

$0.2y = x - 0.7x$

$0.2y = 0.3x$

$x = \frac{0.2y}{0.3}$

$x = \frac{2}{3}y$

From eqn 3

$z = 0.2y + 0.6z$

$0.2y = z - 0.6z$

$0.2y = 0.4z$

$y = \frac{4z}{0.2}$

$y = 2z$

$x = \frac{2}{3}y$ —— ①

$0.4y = 0.3x + 0.4z$ —— ②

$y = 2z$ —— ③

$x + y + z = 1$ —— ④

Substitute $y = 2z$, $x = \frac{2}{3}y$ into eqn 4

$\left(\frac{2}{3}y\right) + (2z) + z = 1$

$\frac{2y + 9z}{3} = 1$

$2y + 9z = 3$ —— ⑤

Input $y = 2z$ into eqn 5

$2(2z) + 9z = 3$

$4z + 9z = 3$

$13z = 3$

$z = \frac{3}{13}$ //

$y = \frac{6}{7}$

If $y = 4/7$

and $x + y = 1$

$x = 3/7$

for

Determine the steady State

put $z = 3/13$ into 3

$y = 2z$

$y = 2 \times \frac{3}{13}$

$y = \frac{6}{13}$

put $y = 6/13$ into 1

$x = \frac{2}{3} y$

$x = \frac{2}{3_1} \times \frac{6^2}{13}$

$x = \frac{4}{13_{11}}$

$\therefore x = 4/13 , y = 6/13 , z = 3/13$

6) a)

Machine	A	B	C
X	(25)	(15)	22
Y	31	20	19
Z	35	24	(17)

Identify the smallest numbers in each column.
Then deduct the smallest number in each column

Machine	A	B	C
X	(0)	0	5
Y	6	5	(2)
Z	10	9	(0)

Identify the smallest number in each row.
Then deduct the smallest number in each row

Machine	A	B	C
X	0	0	5
Y	4	(3)	0
Z	10	9	0

Cross all the zeros with minimal number of lines
Pick the smallest 3 subtract from the rest and add 3 to 5

Machine	A	B	C
X	0	0	8
Y	1	0	0
Z	7	6	0

$\therefore X = C\ 22$

$Y = B\ 20$

$Z = A\ 35$

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