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19/MHS01/300
MBBS
Math 104

1. Integrate the following functions

a) $\int \sin^6 x\,dx$

$\Rightarrow \sin^6 x = (\sin^2 x)^3 (\sin^2 x)$

Recall that: $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\Rightarrow \left(\frac{1-\cos 2x}{2}\right)^3 \left(\frac{1-\cos 2x}{2}\right)$

$\Rightarrow \frac{1}{2}(1-\cos 2x)^3 \cdot \frac{1}{2}(1-\cos 2x)$

$\Rightarrow \frac{1}{8}\int (1-\cos 2x)^3 \cdot (1-\cos 2x)$

$\Rightarrow \frac{1}{8}\int (1-2\cos 2x+\cos^2 2x)\cdot(1-\cos 2x)$

$\Rightarrow \frac{1}{8}\int \left(1-2\cos 2x+\left(\frac{1+\cos 2(2x)}{2}\right)\right)\cdot(1-\cos 2x)$

$\Rightarrow \frac{1}{8}\int \left(1-2\cos 2x+\frac{1+\cos 4x}{2}\right)\cdot(1-\cos 2x)$

$\Rightarrow \frac{1}{16}\int (2-4\cos 2x+1+\cos 4x)(1-\cos 2x)$

$\Rightarrow \frac{1}{16}\int (3-4\cos 2x+\cos 4x)(1-\cos 2x)$

$\Rightarrow \frac{1}{16}\int (3-4\cos 2x+\cos 4x-3\cos 2x+4\cos^2 2x-\cos 4x\cos 2x)$

$\Rightarrow \frac{1}{16}\int (3-7\cos 2x+\cos 4x+(2\cdot 2\cos^2 2x)-\frac{1}{2}\cdot 2\cos 4x\cos 2x)$

$\Rightarrow \frac{1}{16}\int (3-7\cos 2x+\cos 4x+2(1+\cos 4x)-\frac{1}{2}(\cos 6x+\cos 2x))$

$\Rightarrow \frac{1}{32}\int (6-14\cos 2x+2\cos 4x+4+4\cos 4x-\cos 6x-\cos 2x)$

$\Rightarrow \frac{1}{32}\int (10-14\cos 2x+2\cos 4x+4\cos 4x-\cos 6x-\cos 2x)$

$\Rightarrow \frac{1}{32}\int (10-15\cos 2x+6\cos 4x-\cos 6x)\,dx$

$\Rightarrow \frac{1}{32}\left[10x-\frac{15\sin 2x}{2}+\frac{6\sin 4x}{4}-\frac{\sin 6x}{6}\right]+C$

$\Rightarrow \frac{1}{32}\left[10x-\frac{15\sin 2x}{2}+\frac{3\sin 4x}{2}-\frac{\sin 6x}{6}\right]+C$

$\Rightarrow \int \sin^6 x\,dx = \frac{1}{32}\left[10x-\frac{15\sin 2x}{2}+\frac{3\sin 4x}{2}-\frac{\sin 6x}{6}\right]+C$

2. $\int \cos^4 x \sin^3 x \, dx$

$\Rightarrow \int \cos^4 x \cdot \sin^2 x \cdot \sin x \, dx$

Recall that; $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\Rightarrow \int \cos^4 x \cdot \frac{1}{2}(1 - \cos 2x) \cdot \sin x \, dx$

Let; $u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow du = -\sin x \, dx$

$\Rightarrow -du = \sin x \, dx$

$\Rightarrow 2u^2 - 1 = \cos 2x$

$\Rightarrow \frac{1}{2}\int u^4 \cdot (1 - (2u^2 - 1)) \, du$

$\Rightarrow -\frac{1}{2}\int u^4 \cdot (1 - 2u^2 + 1) \, du$

$\Rightarrow -\frac{1}{2}\int u^4 \cdot (2 - 2u^2) \, du$

$\Rightarrow -\frac{1}{2}\int u^4 \cdot 2(1 - u^2) \, du$

$= -\int u^4 (1 - u^2) \, du$

$\Rightarrow -\int u^4 \cdot u^6 \, du$

$\Rightarrow -\frac{u^5}{5} - \frac{u^7}{7} + c$

$\Rightarrow -\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

$\Rightarrow \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + c$

$\Rightarrow \int \cos^4 x \sin^3 x \, dx = \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + c$

3 $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\Rightarrow du = \cos x \, dx$

$\int u^3 \cdot du$

$\Rightarrow \frac{u^{3+1}}{3+1} + c$

$\Rightarrow \frac{u^4}{4} + c$

$= \frac{\sin^4 x}{4} + c$

$\Rightarrow \int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + c$