

1.  $\int \sin^6 x dx$

$$\begin{aligned} &\text{Recall } \sin^2 x = 1 - \cos^2 x \\ &= \int (\sin^2 x)^3 dx \\ &= \int (1 - \cos^2 x)^3 dx \\ &= \int (1 - 2\cos^2 x + \cos^4 x)(1 - \cos^2 x) dx \\ &= \int (1 - \cos^2 x - 2\cos^2 x + 2\cos^4 x + \cos^4 x - \cos^6 x) dx \\ &\rightarrow \int (1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x) dx \\ &= -\frac{3\cos^3 x}{3} - \frac{3\cos^5 x}{5} - \frac{\cos^7 x}{7} + C \\ &= -\cos^3 x - \frac{3\cos^5 x}{5} - \frac{\cos^7 x}{7} + C \end{aligned}$$

2.  $\int \cos^4 x \sin^3 x dx$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{-du}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\begin{aligned} \sin^2 x &= 1 - \cos^2 x \\ &= \sin^2 x \cdot \sin^2 x \cdot \frac{-du}{\sin x} \cdot u^4 \\ &= -\int \sin x \cdot \sin^2 x \cdot u^4 du \\ &= -\int \sin x (1 - \cos^2 x) \cdot u^4 du \\ &= -\int \sin x (1 - u^2) u^4 du \\ &= -\int \sin x (u^4 - u^6) du \\ &= -\cos x \left[ \frac{u^5}{5} - \frac{u^7}{7} \right] + C \\ &= -\cos x \left[ \frac{(\cos x)^5}{5} - \frac{(\cos x)^7}{7} \right] + C \end{aligned}$$

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19/MH501/422

$$\int \cos x \sin 3x \, dx$$

$$\text{Let } A = x, B = 3x$$

$$\text{Recall } \cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$= \frac{1}{2} [\sin(x+3x) - \sin(x-3x)]$$

$$= \frac{1}{2} [\sin 4x - \sin(-2x)]$$

$$= \frac{1}{2} \left[ \sin 4x - \frac{\cos 4x}{4} + \frac{\cos -2x}{-2} \right]$$

$$= \frac{-\cos 4x}{8} + \frac{\cos(-2x)}{-4} + C$$