MAT 104 ASSIGNMENT

Integrate the following functions:

1) $\sin^6 x$

2) $\cos^4 x \sin^3 x$

3) $\cos x \sin 3x$

Solution

~~1 $\int \sin^6 x\,dx = \int \sin x \cdot \sin^5 x\,dx$~~

1. $\int \sin^6(x)\,dx = \int (\sin^2(x))^3\,dx$

Recall:

$$\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta))$$

$$\int(\sin^2(x))^3 dx = \int\left(\frac{1}{2}(1-\cos(2x))\right)^3 dx$$

$$= \int \left(\frac{1}{2}\right)^3 (1-\cos(2x))^3\,dx$$

$$= \int \frac{1}{8}(1-\cos(2x))^3\,dx$$

$$= \frac{1}{8}\int\left(1-3\cos(2x)+3\cos^2(2x)-\cos^3(2x)\right)dx$$

$$= \frac{1}{8}\int dx - \frac{3}{8}\int\cos(2x)\,dx + \frac{3}{8}\int\cos^2(2x)\,dx - \frac{1}{8}\int\cos^3(2x)\,dx$$

$$= \frac{x}{8} - \frac{3}{8}\times\frac{\sin(2x)}{2} + \frac{3}{8}\left(\frac{x}{2}+\frac{\sin(4x)}{8}\right) - \frac{1}{8}\left(\frac{\sin(2x)}{2} - \frac{\sin^3(2x)}{6}\right) + c$$

$$= \frac{x}{8} - \frac{3\sin(2x)}{16} + \frac{3x}{16} + \frac{3\sin(4x)}{64} - \frac{\sin(2x)}{16} + \frac{\sin^3(2x)}{48} + c$$

$$= \frac{x}{8} + \frac{3x}{16} - \frac{3}{16}\sin(2x) + \frac{3}{64}\sin(4x) - \frac{1}{6}\sin(2x) + \frac{1}{48}\sin^3(2x) + c$$

$$= \frac{5(x)}{16} - \frac{3}{16}\sin(2x) + \frac{3}{64}\sin(4x) - \frac{1}{6}\sin(2x) + \frac{1}{48}\sin^3(2x) + c$$

2. $\int \cos^4 x \sin^3 x\,dx = \int \sin(x)\cdot\sin^2(x)\cdot\cos^4(x)\,dx$

$$= \int \sin(x)\left[1-\cos^2(x)\right]\cdot\cos^4(x)\,dx$$

Let $u = \cos(x)$

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

$$= \int \sin(x)(1-u^2)u^4\,dx = \int(1-u^2)u^4\sin(x)\,dx$$

$$= \int (1-u^2)\,u^4 \cdot -du$$

$$= \int (u^6 - u^4)\,du = \frac{u^7}{7} - \frac{u^5}{5} + c$$

$$\frac{-\cos^7(x)}{7} - \frac{\cos^5(x)}{5} + c$$

3) $\int \cos(x)\sin(3x) = \int \sin(3x)\cos(x)\,dx$

Recall;

$$\sin a \cdot \cos b = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$$

$$\sin(3x)\cos(x) = \frac{1}{2}\left(\sin(4x) + \sin(2x)\right)$$

$$\int \sin(3x)\cos(x) = \frac{1}{2}\int \sin(4x)\,dx + \frac{1}{2}\int \sin(2x)\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos(4x)}{4}\right) + \frac{1}{2}\left(-\frac{\cos(2x)}{2}\right) + c$$

$$= -\frac{\cos(4x)}{8} - \frac{\cos(2x)}{4} + c$$