

MAT 104 ASSIGNMENT

Integrate the following functions:

1)  $\sin^6 x$

2)  $\cos^4 x \sin^3 x$

3)  $\cos x \sin 3x$

solution

1.  $\int \sin^6 x dx = \int \sin^2 x \cdot \sin^4 x dx$

$\int \sin^6(x) dx = \int (\sin^2(x))^3 dx$

Recall:

$$\sin^2(\theta) = \frac{1}{2} (1 - \cos(2\theta))$$

$$\int (\sin^2(x))^3 dx = \int \left( \frac{1}{2} (1 - \cos(2x)) \right)^3 dx$$

$$= \int \left( \frac{1}{2} \right)^3 (1 - \cos(2x))^3 dx$$

$$= \int \frac{1}{8} (1 - \cos(2x))^3 dx$$

$$= \frac{1}{8} \int (1 - 3\cos(2x) + 3\cos^2(2x) - \cos^3(2x)) dx$$

$$= \frac{1}{8} \left( \int dx - 3 \int \cos(2x) dx + 3 \int \cos^2(2x) dx - \int \cos^3(2x) dx \right)$$

$$= \frac{x}{8} - \frac{3x \sin(2x)}{8} + \frac{3}{8} \left( \frac{x}{2} + \frac{\sin(4x)}{8} \right) - \frac{1}{8} \left( \frac{\sin(2x)}{2} - \frac{\sin^3(2x)}{6} \right) + C$$

$$= \frac{x}{8} - \frac{3 \sin(2x)}{16} + \frac{3x}{16} + \frac{3 \sin(4x)}{64} - \frac{\sin(2x)}{16} + \frac{\sin^3(2x)}{48} + C$$

$$= \frac{x}{8} + \frac{3x}{16} - \frac{3 \sin(2x)}{16} + \frac{3 \sin(4x)}{64} - \frac{1 \sin(2x)}{16} + \frac{1 \sin^3(2x)}{48} + C$$

$$= \frac{5x}{16} - \frac{3 \sin(2x)}{16} + \frac{3 \sin(4x)}{64} - \frac{1 \sin(2x)}{16} + \frac{1 \sin^3(2x)}{48} + C$$

2.  $\int \cos^4 x \sin^3 x dx = \int \sin(x) \cdot \sin^2(x) \cdot \cos^4(x) dx$

$$= \int \sin(x) [1 - \cos^2(x)] \cdot \cos^4(x) dx$$

Let  $u = \cos(x)$

$$du = -\sin(x) dx$$

$$-du = \sin(x) dx$$

$$= \int \sin(x) (1 - u^2) u^4 dx = \int (1 - u^2) u^4 \sin(x) dx$$

$$\begin{aligned}
 &= \int (1-u^2)u^4 \cdot -du \\
 &= \int (u^6 - u^4) du = \frac{u^7}{7} - \frac{u^5}{5} + C \\
 &= \frac{-\cos^7(x)}{7} - \frac{\cos^5(x)}{5} + C
 \end{aligned}$$

$$3) \int \cos(x) \sin(3x) = \int \sin(3x) \cos(x) dx$$

Recall,

$$\sin a \cdot \cos b = \frac{1}{2} (\sin(a+b) + \sin(a-b))$$

$$\sin(3x) \cos(x) = \frac{1}{2} (\sin(4x) + \sin(2x))$$

$$\int \sin(3x) \cos(x) = \frac{1}{2} \int \sin(4x) dx + \frac{1}{2} \int \sin(2x) dx$$

$$= \frac{1}{2} \left( \frac{-\cos(4x)}{4} \right) + \frac{1}{2} \left( \frac{-\cos(2x)}{2} \right) + C$$

$$= -\frac{\cos(4x)}{8} - \frac{\cos(2x)}{4} + C$$