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**A REVISED REPORT**

**ON**

**ELECTRICAL CONDUCTIVITY OF FLUID-SATURATED ROCKS**

**AN EXTRACT FROM**

**PETROLEUM RESERVOIR ENGINEERING PHYSICAL PROPERTIES**

**BY**

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ABSTRACT

The whole of resistivity logging is based upon a few very important equations which are introduced in this section. The equations, which are known as the Archie Equations, relate the resistivity of a formation to the resistivity of the fluids saturating a formation, the porosity of the formation and the fractional degree of saturation of each fluid present.

Porous media consist of mineral, rock fragments and void space. The solids with exception of certain clay minerals (such as shaly sands where clay shales produce electrical conductivity) are nonconductive. Generally the electrical property of a rock depends on void space geometry and the fluids that occupy the void space. The fluid of interest for petroleum engineers are oil, gas and water. Oil and gas are nonconductor and water is conductive when contains dissolved salts.  Water has electrolyte conductivity, because electricity conducted by movement of ions.

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CHAPTER ONE

# **UNDERSTANDING ELECTRICAL CONDUCTIVITY**

Measuring the electrical resistivity of the subsurface is the most powerful prospecting method in geothermal exploration. Resistivity is directly related to the properties of interest, like salinity, temperature, porosity (permeability) and alteration. To a great extent, these parameters characterize the reservoir. Resistivity can also be defined as the ratio of the potential difference, $∆V({V}/{m})$, to the current, $I(A)$, across material which has a cross-sectional area of 1 $m^{2}$ and is 1 m long.

$$ρ=\frac{∆V}{I}$$

Electrical conductivity in minerals and solutions takes place by movement of electrons and ions. Most rocks near the earth’s surface have low conductivity. Conduction of electricity is mostly through groundwater contained in pores of the rocks and along surface layers at the contact of rocks and solution. The case of altered rocks in high temperature areas will be discussed separately.

## **RESISTIVITY RELATIONS**

Porous rocks are comprised of an aggregate of minerals, rock fragments, and void spaces. The electrical properties predominantly depend on the geometry of these void spaces and the fluids which resides in the void. In the study of petroleum engineering we are concerned with three fluids they are oil, gas and water. Oil and gas are nonconductors while water is a conductor. This is due to the presence of mobile ions that allows the flow of current, this is usually termed as electrolytic conduction. By knowing and estimating the resistivity of the rock matrix, we can further determine its conductivity- why?. This is because the reciprocal of resistivity is conductivity.

$$ρ=\frac{rA}{L}$$

Where $ρ=resistivity$

 $r=resistance$

 $A=cross-sectional area of the conductor$

 $L=length of the conductor$

In oil-field practice, the resistivity in ohms-meter is commonly represented by symbol R with an appropriate subscript to define the conditions to which R applies.

## **FORMATION FACTOR**

In the late 1920s and early 1930s Archie carried out a series of experiments to analyze the relationship between the resistivity of rock samples saturated with various resistivity fluids. He used clean sandstones and carbonates which were 100% saturated with aqueous solutions of varying concentration and hence resistivity. His results are empirical (derived from experimental work on particular samples) and hence should not be assumed to have theoretical rigor. However, we are now beginning to see how the theory of the electrical properties of rocks can give rise to the Archie equations. Archie observed that the bulk resistivity of a rock $R\_{o}$ fully saturated with an aqueous fluid of resistivity $R\_{w}$ is directly proportional to the resistivity of the fluid

$$R\_{o} α FR\_{w}$$

Where F is known as the “**formation factor**” or the “**formation resistivity factor**” and describes the effect of the presence of the rock matrix. It can be immediately seen that F = 1.00 for a rock with 100% porosity, i.e., no matrix, just 100% fluid.

The formation resistivity factor, FR, is the ratio of the resistivity of a porous medium that is completely saturated with an ionic brine solution divided by the resistivity of the brine.

$$F=\frac{R\_{o}}{R\_{w}}$$

Where:

* *Ro* = Resistivity of fully water saturated rock
* *Rw* = Saturating water resistivity

The relationship between electrical properties and the physical properties of the rock are complex but can illustrated by the following proof:

CASE 1: Consider matrix cube of salt water having cross sectional area A, length L, and a resistivity of Rw. Suppose an electrical current is caused to flow the cube, the resistivity of the cube can be determined by;

$$r\_{1}=\frac{R\_{w}L}{A}$$

CASE 2: Now let another cube, cube B with the same dimensions as the former cube, cube A of salt water having 100% saturation of water, but cube B is filled with non-conducting solids and partially saturated with water. Consider, electrical current must then flow through the water pores of cube B having a cross sectional area, Aa of the conduction and the length of the current flow La. Finally, the resistance of Cube B is given as;

$$r\_{2}=\frac{R\_{w}L\_{a}}{A\_{a}}$$

Recall

$$R\_{o}=\frac{r\_{2}A}{L}$$

Therefore

$$R\_{o}=\frac{R\_{w}L\_{a}A}{LA\_{a}}$$

But

$$F=\frac{R\_{o}}{R\_{w}}=\frac{{L\_{a}}/{L}}{{A\_{a}}/{A}}=\frac{\sqrt{τ}}{{A\_{a}}/{A}}$$

Where τ is tortuosity

## **RESISTIVITY INDEX**

CASE 3: let consider the cube of a porous rock that contains both hydrocarbons and water, being the only conductor. A decrease in cross sectional area of the conduction is denoted by $A\_{a}^{'}$ and the path length of the current flow to be $L\_{a}^{'}$. Then the resistance of such cube is given as;

$$r\_{3}=\frac{R\_{w}L\_{a}^{'}}{A\_{a}^{'}}$$

Then the resistivity of the entire rock formation is;

$$R\_{t}=\frac{r\_{3}L}{A}=\frac{R\_{w}AL\_{a}^{'}}{LA\_{a}^{'}}$$

The ratio of the resistivity of partially saturated rock to the resistivity of the entire rock is known as resistivity index, I

$$I=\frac{R\_{t}}{R\_{o}}=\frac{{A\_{a}}/{A\_{a}^{'}}}{{L\_{a}}/{L\_{a}^{'}}}$$

From the equations above it is clear that the formation factor and the resistivity index both depends on the cross sectional area and the path length of current, thus its essential to relate these quantities to other physical properties of the rock. Models have been introduced to develop a relation between formation factor $F$, resistivity index $I$, with porosity$ ϕ$ and tortuosity $τ$. These models include;

1. Wyllie model: its considered that the various pore openings are continuous. The cross sectional area of these pore openings vary with their length but in a manner that the sum of areas of the pores is constant.

$$A\_{a}=A\_{1}+A\_{2}=ϕA$$

Therefore

$$F=\frac{{L\_{a}}/{L}}{{ϕA}/{A}}=\frac{L\_{a}}{L}\frac{1}{ϕ}$$

Introducing hydrocarbons to the pores, the water saturation $S\_{w}$ is expressed as a fraction of pore volume. The presence of hydrocarbon further reduced the cross sectional area for current flow to $A\_{a}^{'}$ and the effective path length to $L\_{a}^{'}$

$$A\_{a}^{'}=ϕS\_{w}A$$

Thus

$$I=\frac{{ϕA}/{ϕS\_{w}A}}{{L\_{a}}/{L\_{a}^{'}}}=\frac{L\_{a}^{'}}{L\_{a}}\frac{1}{S\_{w}}$$

1. Cornell & kartz: In this model, the pores can be uniformed in cross sections but oriented so that they have an effective length $L\_{a}$ which is greater than $L$. The cross sectional area available for flow is uniform across the matrix. The effective cross sectional area $A\_{a}$ is the area normal to the direction of the flow of the pore;

$$A\_{a}=A\_{1}^{'}=ϕA\frac{L}{L\_{a}}$$

$$thus the formation factor, F=\frac{{L\_{a}}/{L}}{{φA\left({L}/{L\_{a}}\right)}/{A}}=\left(\frac{L\_{a}}{L}\right)^{2}\frac{1}{ϕ}=\frac{τ}{ϕ}$$

Introducing the hydrocarbon saturation;

$$A\_{a}^{'}=A\_{1}^{''}$$

$$but, A\_{1}^{''}=A\_{1}\frac{L}{L\_{a}^{'}}$$

$$since A\_{a}^{'}=ϕS\_{w}A\frac{L}{L\_{a}^{'}}$$

$$Thus I=\frac{{ϕA\left({L}/{L\_{a}}\right)}/{ϕS\_{w}A\left({L}/{L\_{a}^{'}}\right)}}{{L\_{a}}/{L\_{a}^{'}}}=\left(\frac{L\_{a}^{'}}{L\_{a}}\right)^{2}\frac{1}{S\_{w}}$$

1. Wyllie & Gardner: in this model, the effective cross section is only the net exit at each plane. While the cross sectional area of the pore is uniform again. Understanding that the probability of hitting a pore opening in the continous plane is $ϕ^{2}$

$$A\_{a}=\left(ϕ\right)^{2}A$$

Thus

$$F=\frac{{L\_{a}}/{L}}{{ϕ^{2}A}/{A}}=\frac{L\_{a}}{L}\frac{1}{ϕ^{2}}$$

Recall, $L=L\_{a}$

$$F=\frac{1}{ϕ^{2}}$$

In presence of hydrocarbon

$$A\_{a}^{'}=\left(ϕS\_{w}\right)^{2}A$$

$$L\_{a}^{'}=L\_{a}=L$$

Resistivity index;

$$I=\frac{{\left(ϕ\right)^{2}A}/{ϕ^{2}S\_{w}^{2}A}}{{L\_{a}}/{L\_{a}^{'}}}=\frac{1}{S\_{w}^{2}}$$

In a nutshell, the formation factor is a function of porosity and internal geometry of the rock

$$F=Cϕ^{-m}$$

Where C is a function of tortuosity and m is the number of reduction in pore opening sizes. Since C is a function of the ratio ${L\_{a}}/{L}$, its suggested that C is $\geq 1$. The value of m ranges from 1-2

Archie in 1942, correlated several formation factors with porosity and permeability. He suggested that the formation factor with the porosity was the better correlation and that formation factor should be expressed as

$$F=ϕ^{-m}$$

Where $ϕ$ is the fractional porosity and m is the cementation factor. For consolidated sandstones the cementation factor ranged from 1.8-2.0, while for unconsolidated sandstones is 1.3



Other correlation/model of formation resistivity are in the table below

|  |  |
| --- | --- |
| Source | Relation |
| * Wyllie and rose
 | $$F=\frac{r^{^{1}/\_{2}}}{ϕ}$$ |
| * Tixier
 | $$F=\frac{1}{ϕ^{2}}$$ |
| * Winsauer et al
 | $$F=\frac{r^{2}}{ϕ}$$ |
| * Owen
 | $$F=0.68ϕ^{-2.23}$$ |

CHAPTER TWO

# **MEASUREMENT OF ELECTRICAL RESISTIVITY OF ROCK**

Various devices have been employed to measure the electrical resistivity of rock. The measurement requires a knowledge of the dimension of rock, fluid saturation of rock, the resistivity of the water contained in the rock, and a suitable resistivity cell in which the test is carried out.

A sample cut of the rock is placed in the cell and clamped between electrodes, current is then passed through the sample and the potential drop is observed and the resistance is calculated from ohms law;

$$r=\frac{I}{E}$$

And the resistivity is calculated from;

$$R=\frac{rA}{L}$$

Where A is the cross sectional area of the sample, L is the length of the sample

## **EMPIRICAL CORRELATION OF ELECTRICAL PROPERTIES**

Some of these correlation have been discussed earlier and a short table gives an overview. These correlations include;

* Stawinski and Maxwell developed models for unconsolidated spheres. Stawinski for spheres that are in contacts:

$$F=\frac{\left(1.3219-0.3219ϕ\right)^{2}}{ϕ}$$

While for dispersed spheres, Maxwell states that;

$$F=\frac{3-ϕ}{2-ϕ}$$

* Wyllie was concerned with the influence of particle size and cementation on the formation factor of a variety of materials. Unconsolidated materials were packed in tubes, and were artificially consolidated. It may be noted that cementation results in increased values of formation factor over that observed for un-cemented aggregates. Furthermore, the cemented aggregates exhibit a greater change in formation factor with a change in porosity than unconsolidated aggregates. The correlation include;

$$F=Cϕ^{-m}$$

$$where C is a constant controlled by the porosity, m is a constant depending on cementation$$

* Winsaeur reported a correlation referred to as the Humble relations

$$F=0.62ϕ^{-2.15}$$

## **EFFECT OF CONDUCTIVE SOLIDS**

The formation factor for a clean sand is constant. The formation factor for the clayey sand increases with decreasing water resistivity and approach a constant value at a water resistivity of about 0.1ohm-m. This phenomenon is similar to clay minerals in two electrical circuits connected in parallel

$$F=\frac{R\_{oa}}{R\_{w}} and \frac{1}{R\_{oa}}=\frac{1}{R\_{o}}+\frac{1}{FR\_{w}}$$

Where $R\_{oa}$ is the resistivity of a shaly sand when 100% saturated with water resistivity $R\_{w}$, $R\_{o}$ is the resistivity due to the clay minerals; $FR\_{w}$ is the resistivity due to distributed water and F is the true formation factor of the rock.

Hill and Milburn developed a correlation for the formation factor of shaly sand. It was based on the measurement of large number of samples

$$F\_{a}=F\_{0.01}R\_{w}^{b log⁡\left(100R\_{w}\right)}$$

Where $F\_{0.01}$ is the formation factor of the rock when saturated with water having resistivity of 0.01ohm-m. The quantity $b$ in the exponent is defined as a shaliness factor and was correlated with cation-exchange capacity. The cation-exchange capacity is related to the clay content of the rock and provides an independent determination of the amount of shale in the rock

CHAPTER THREE

# RESISTIVITY OF PARTIALLY WATER-SATURATED ROCK

From studies, a rock containing both hydrocarbon and water has a higher resistivity than another that is fully saturated with water. The resistivity of partially water-saturated rock is a function of the water saturation $S\_{w}$

Meanwhile the resistivity index is a function of the water saturation and path length, thus the following axiom is produced;

$$I=C^{'}S\_{w}^{-n}$$

Where C is a function of tortuosity and n is saturation exponent. Few scientist made improvement to the general;

* Achie - $I=S\_{w}^{-2}$
* Willams - $I=S\_{w}^{-2.7}$

## **USE OF ELECTRICAL PARAMETER IN CHARACTERIZING POROUS MEDIA**

This entails kozeny equation and it is developed as follows;

$$k=\frac{ϕ}{k\_{0}τS\_{p}^{2}}$$

where $k$ is the permeability, $ϕ$ is the porosity function, $k\_{0}$ is a shape factor, $S\_{p}$ is the internal surface area per unit pore volume and $τ$ is the kozeny tortuosity.

CHAPTER FOUR

# CONCLUSION

In a nutshell, it can be shown that the following variables have effects on the resistivity of natural porous media:

* Temperature
* Porosity
* Pore geometry
* Formation stress
* Rock composition
* Degree of cementation
* Type of pore system inter-crystalline
* Sorting and Packing (in particulate system)

The last six factors have effect through the influence on the conduction path. Confinement or overburden pressure may cause a significant increase in resistivity by blocking of some conduction paths and reduction in the cross sections which are available for flow.  This usually occurs in rocks with low porosity or that are not well cemented. The formation resistivity factor *F*, which is sometimes referred to as the formation factor, can be estimated from an empirical relationship between formation resistivity factor *F* and porosity ϕ. The empirical relationship is $F=aϕ^{-m}$

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