

MBBS Assignment 19/mh011235 LEBILE CERANE-M.

2. $\int \cos^4 x \sin^3 x \, dx$

Let $u = \cos x \quad \therefore \frac{du}{dx} = -\sin x$

$dx = -\frac{du}{\sin x}$

$\int u^4 \cdot \sin x \cdot \sin^2 x \cdot \frac{du}{\sin x}$

$\int u^4 \cdot (1 - \cos^2 x) \cdot du$

$\int (1 - \cos^2 x) \cdot u^4 \, du$

$\int (1 - u^2) \cdot u^4 \, du$

$\int (u^6 - u^4) \, du$

$\left[\frac{u^{6+1}}{6+1} - \frac{u^{4+1}}{4+1} \right] + C$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$\left[\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C = \int \cos^4 x \sin^3 x \, dx \right]$

3. $\int \cos x \sin^3 x \, dx$

Let $u = \sin x \quad \frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$

$\int u^3 \, du$

$\frac{u^{3+1}}{3+1} = \frac{u^4}{4} + C$

$\left[\frac{\sin^4 x}{4} + C = \int \cos x \sin^3 x \, dx \right]$

$$4 \int \sin^6 x \, dx = \int [\sin^2 x]^3 \, dx$$

$$= \int \left[\frac{1 - \cos 2x}{2} \right]^3 \, dx$$

$$\int \frac{1}{8} (1 - \cos 2x)^3 \, dx$$

$$\text{But } (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$\therefore \frac{1}{8} \int [1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x] \, dx$$

$$\frac{1}{8} \int dx - 3 \int \cos 2x + 3 \int \cos^2 2x - \int \cos^3 2x$$

$$\frac{1}{8} \left[x - \frac{3\sin 2x}{2} + 3 \int \frac{1 + \cos 4x}{2} \right] \quad \left. \begin{array}{l} \text{let } u = \sin 2x \\ dx = \frac{du}{2\cos 2x} \end{array} \right\}$$

$$\frac{3}{2} \int \frac{1}{2} (1 + \cos 4x) \quad \left. \begin{array}{l} - \int \cos 2x (1 - \sin^2 2x) \frac{du}{2\cos 2x} \\ - \int \frac{1}{2} (1 - u^2) \, du \end{array} \right\}$$

$$\frac{3}{2} \left[\frac{x}{4} + \frac{\sin 4x}{4} \right]$$

$$\frac{3}{2} \left[\frac{x}{2} + \frac{3\sin 4x}{8} \right]$$

$$\left[-\frac{1}{2} \left(u - \frac{u^3}{3} \right) + C \right]$$

$$-\frac{u}{2} + \frac{u^3}{6} + C$$

$$-\frac{\sin 2x}{2} + \frac{\sin^3 2x}{6} + C$$

$$-\frac{\sin 2x}{2} + \frac{\sin^3 2x}{6} + C$$

$$\therefore \frac{1}{8} \left[x - \frac{3\sin 2x}{2} + \frac{3x}{2} + \frac{3\sin 4x}{8} - \frac{\sin 2x}{2} + \frac{\sin^3 2x}{6} \right] + C$$

$$= \frac{1}{8} \left[\frac{x}{16} - \frac{3\sin 2x}{16} + \frac{3x}{16} + \frac{3\sin 4x}{64} - \frac{\sin 2x}{16} + \frac{\sin^3 2x}{48} \right] + C$$

$$\frac{1}{8}x + \frac{3x}{16} - \frac{3 \sin 2x}{16} - \frac{\sin 2x}{16} + \frac{3 \sin 4x}{64} + \frac{\sin^3 2x}{48} + C$$

$$\left(\frac{1}{8} + \frac{3}{16}\right)x - \left(\frac{3}{16} + \frac{1}{16}\right) \sin 2x + \frac{3 \sin 4x}{64} + \frac{\sin^3 2x}{48} + C$$

$$\frac{5x}{16} - \frac{4 \sin 2x}{16} + \frac{3 \sin 4x}{64} + \frac{\sin^3 2x}{48} + C$$