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MATRIC NO: 19/MHS01/147

COLLEGE: MEDICINE AND HEALTH SCIENCES

DEPARTMENT: MEDICINE AND SURGERY

COURSE CODE: MAT 104

MAT 104 ASSIGNMENT

1 $\int \sin^6 x \, dx = \int (\sin^4 x)^3 \, dx$

$(\sin^2 x)^3 = \left(\dfrac{1-\cos 2x}{2}\right)^3$

$= \dfrac{1}{8}\int (1-\cos 2x)^3 \, dx$

$= \dfrac{1}{8}\int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$

$\cos^2 2x = \dfrac{1+\cos 2(2x)}{2} = \dfrac{1+\cos 4x}{2}$

$\cos^3 2x = \cos^2 2x \cdot \cos 2x$

Let $\cos^2 2x = 1 - \sin^2 2x$

$= (1-\sin^2 2x)\cos 2x$

$= \cos 2x - \sin^2 2x \cos 2x$

$\therefore \dfrac{1}{8}\int 1 - 3\cos 2x + 3\left(\dfrac{1}{2} + \dfrac{\cos 4x}{2}\right) - (\cos 2x - \sin^2 2x \cos 2x) dx$

$= \dfrac{1}{8}\int 1 - 3\cos 2x + \dfrac{3}{2} + \dfrac{3\cos 4x}{2} - \cos 2x + \sin^2 2x \cos 2x \, dx$

$= \dfrac{1}{8}\int \dfrac{5}{2} - 4\cos 2x + \dfrac{3\cos 4x}{2} + \sin^2 2x \cos 2x \, dx$

$\int \sin^2 2x \cos 2x \, dx$

Let $u = \sin 2x$; $\dfrac{du}{dx} = 2\cos 2x$; $du = 2\cos 2x \, dx$

$\Rightarrow \int u^2 \dfrac{du}{2} = \dfrac{1}{2}\int u^2 \, du$

$\Rightarrow \dfrac{1}{2}\left[\dfrac{u^3}{3}\right] + C = \dfrac{\sin^3 2x}{6} + C$

$$\therefore \frac{1}{8}\left[\frac{5}{2}x - \frac{\cancel{4}^{2}\sin 2x}{\cancel{2}_{1}} + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6}\right] + C$$

$$= \frac{5}{16}x - \frac{\cancel{2}^{1}\sin 2x}{\cancel{8}_{4}} + \frac{3\sin 4x}{64} + \frac{\sin^3 2x}{48} + C$$

$$= \frac{5}{16}x - \frac{\sin 2x}{4} + \frac{3\sin 4x}{64} + \frac{\sin^3 2x}{48} + C$$

2 $\cos^4 x \sin^3 x$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$; $dx = \frac{du}{-\sin x}$

Recall

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$\int \cos^4 x \cdot \sin^2 x \cdot \sin x \, dx$$

$$= \int u^4 \cdot \sin^2 x \cdot \sin x \frac{du}{-\sin x}$$

$$= -\int u^4 \cdot \sin^2 x \cdot \cancel{\sin x} \cdot \frac{du}{\cancel{\sin x}}$$

$$= -\int u^4 (1 - \cos^2 x)\, du$$

$$= -\int u^4 (1 - u^2)\, du$$

$$= -\int (u^4 - u^6)\, du$$

$$= \int (u^6 - u^4)\, du$$

$$= \left[\frac{u^7}{7} - \frac{u^5}{5}\right] + C$$

$$\frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

$$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

3 $\displaystyle\int \cos x \sin^3 x \, dx$

Let $u = \cos x$; $\frac{du}{dx} = -\sin x$; $dx = \frac{du}{-\sin x}$

Recall

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$= \int u \cdot \sin^2 x \cdot \sin x \frac{du}{-\sin x}$$

$$= -\int u \cdot \sin^2 x \cdot \cancel{\sin x} \frac{du}{\cancel{\sin x}}$$

$$= -\int u(1 - \cos^2 x) \, du$$

$$= -\int u(1 - u^2) \, du$$

$$= -\int (u - u^3) \, du$$

$$= \int (u^3 - u) \, du \Rightarrow \frac{u^4}{4} - \frac{u^2}{2} + C$$

$$\Rightarrow \frac{u^4}{4} - \frac{u^2}{2} + C$$

$$= \frac{(\cos x)^4}{4} - \frac{(\cos x)^2}{2} + C$$

$$= \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C$$