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1. Integration of given function $\sin^6 x$

SOLUTION

$$\int \sin^6(x)\,dx$$

Apply reduction formula

$$\int \sin^n(x)\,dx = \frac{n-1}{n}\int \sin^{n-2}(x)\,dx - \frac{\cos(x)\sin^{n-1}(x)}{n}$$

with $n=6$

$$-\frac{\cos(x)\sin^5(x)}{6} + \frac{5}{6}\int \sin^4(x)\,dx$$

or choose an alternative (product to sum formulas)

$$\int \sin^4(x)\,dx$$

Apply the last reduction formula again with $n=4$

$$-\frac{\cos(x)\sin^3(x)}{4} + \frac{3}{4}\int \sin^2(x)\,dx$$

or choose an alternative (apply product to sum formulas)

$$\int \sin^2(x)\,dx$$

Apply the last reduction formula again with $n=2$

$$-\frac{\cos(x)\sin(x)}{2} + \frac{1}{2}\int 1\,dx$$

or choose an alternative (product to sum formulas)

$$\int 1\,dx$$

Apply constant rule

$= x$

Add with solved integrals

$$-\frac{\cos(x)\sin(x)}{2} + \frac{1}{2}\int 1\,dx$$

$$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$$

Add in solved integrals

$$-\frac{\cos(x)\sin^3(x)}{4} + \frac{3}{4}\int \sin^2(x)\,dx$$

$$-\frac{\cos(x)\sin^3(x)}{4} - \frac{3\cos(x)\sin(x)}{8} + \frac{3x}{8}$$

Add in solved integrals

$$-\frac{\cos(x)\sin^5(x)}{6} + \frac{5}{6}\int \sin^4(x)\,dx$$

$$-\frac{\cos(x)\sin^5(x)}{6} - \frac{5\cos(x)\sin^3(x)}{24} - \frac{5\cos(x)\sin(x)}{16} + \frac{5x}{16}$$

$\therefore$ f

$$= -\frac{\cos(x)\sin^5(x)}{6} - \frac{5\cos(x)\sin^3(x)}{24} - \frac{5\cos(x)\sin(x)}{16} + \frac{5x}{16} + C$$

$$= -\frac{\sin(6x) - 9\sin(4x) + 45\sin(2x) - 60x}{192} + C$$

2. Inte

$= \int$

2. Integration of given function $\cos^4 x \sin^3 x$

SOLUTION

$$\int \cos^4(x)\sin^3(x)\,dx$$

Substitution, use $\sin^2(x) = 1 - \cos^2(x)$

$$= \int -\cos^4(x)(\cos^2(x) - 1)\cdot\sin(x)\,dx$$

Substitute $u = \cos(x) \rightarrow du/dx = -\sin(x)$

$$\rightarrow dx = -\frac{1}{\sin(x)}\,du$$

$$= \int u^4(u^2 - 1)\,du$$

Choose an alternative (Don't substitute)

Expand

$$= \int (u^6 - u^4)\,du$$

Apply linearity

$$= \int u^6\,du - \int u^4\,du$$

$$\int u^6\,du$$

Apply power rule

$$\int u^n\,du = \frac{u^{n+1}}{n+1} \quad \text{with } n = 6$$

$$= \frac{u^7}{7}$$

$$\int u^4\,du$$

Apply power rule with $n = 4$

$$= u^5/5$$

Add in solved integrals

$$\int u^6\,du - \int u^4\,du$$

$$= \frac{u^7}{7} - \frac{u^5}{5}$$

Undo substitution $u = \cos(x)$

$$= \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$$

$$\int \cos^4(x)\sin^3(x)\,dx$$
$$= \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5} + C$$
$$= \frac{5\cos^7(x) - 7\cos^5(x)}{35} + C$$

3. Integration of given function $\cos x \sin^3 x$

SOLUTION

$$\int \cos(x)\sin^3(x)\,dx$$

Substitue $u = \sin(x) \rightarrow du/dx = \cos(x)$

$\longrightarrow dx = \frac{1}{\cos(x)}\,du$:

$$= \int u^3\,du$$

Apply power rule:

$\int u^n du = \frac{u^{n+1}}{n+1}$ with $n = 3$:

$$= \frac{u^4}{4}$$

Undo substitution $u = \sin(x)$

$$= \frac{\sin^4(x)}{4}$$

$$\int \cos(x)\sin^3(x)\,dx$$
$$= \frac{\sin^4(x)}{4} + C$$