1 $\sin^6 x$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$

$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1 - \cos 2x)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1 - \cos x)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1 - \cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x$

$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \frac{1}{2} \cdot 2\cos 4x \cos$

$= \frac{1}{16}\{3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x] - \frac{1}{2}(\cos 6x + \cos$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)]\}$

$= \frac{1}{32}[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos$

$= \frac{1}{32}[10 - 15\cos 2x + 6\cos 4x - \cos 6x]$

So,

$I = \frac{1}{32}\left[10x - \frac{15\sin x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + C$

$I = \frac{1}{32}\left[10x - \frac{15\sin x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + C$

$I = \frac{1}{192}[60x - 45\sin 2x + 9\sin 4x - \sin 6x] + C$,

2. $\sin^3 x \cos^4 x \, dx = -\frac{1}{5}\cos^5(x) + \frac{1}{7}\cos^7(x) + C$

$\int \sin^3(x) \cos^4(x) dx$

$\sin^3(x)\cos^4(x) = (1 - \cos^2(x)] \cos^4(x) \sin(x)$

$= \cos^4(x)\sin(x) - \cos^6(x)\sin(x)$

then

$\int \sin^3(x) \cos^4(x) \, dx = \int \cos^4(x) \sin(x) - \cos(x)$

$= -\frac{1}{5}\cos^5(x) + \frac{1}{7}\cos^7(x) + C.$

3 $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$dx = du / \cos x.$

$$\int \cos x \sin^3 x \, dx = \int \cancel{\cos x} \, u^3 \frac{du}{\cancel{\cos x}} = \int u^3 \, du$$

$$\int \cos x \sin^3 x \, dx = \frac{u^4}{4} + c = \frac{\sin^4 x}{4} + c$$