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19/MHS01/101

MATHS 104

Integration

1 $\int \sin^6 x \, dx$

Solution

$\int \sin^6 x = \int \sin^2 x \cdot \sin^4 x \, dx$

$\int \sin^6 x = \int \frac{1-\cos 2x}{2} \cdot \left(\frac{1-\cos 2x}{2}\right)^2 dx$

$\int \sin^6 x = \int \frac{1-\cos 2x}{2} \cdot \frac{1-2\cos 2x+\cos^2 2x}{4} \, dx$

$\int \sin^6 x = \frac{1}{8}\int 1-2\cos 2x+\cos^2 2x-\cos 2x+2\cos^2 2x-\cos^3 2x \, dx$

$\int \sin^6 x = \frac{1}{8}\int 1-3\cos 2x+3\cos^2 2x-\cos^3 2x \, dx$

$\int \sin^6 x = \frac{1}{8}\int 1-3\cos 2x+3\left(\frac{1+\cos 4x}{2}\right)-\cos 2x(1-\sin^2 x) \, dx$

$\int \sin^6 x = \frac{1}{8}\int 1-3\cos 2x+\frac{3}{2}+\frac{3\cos 4x}{2}-\cos 2x+\cos 2x \sin^2 x \, dx$

$$\int \sin^6 x = \frac{1}{8}\int \frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x \, dx$$

$$\cancel{\int \sin 6x = \frac{1}{8}\left[\frac{5}{2}x - \frac{4\sin 2x}{2} + \frac{3\sin 4x \times 4}{2}\right.}$$

$$\int \sin 6x = \frac{1}{8}\left[\frac{5}{2}x - \frac{4\sin 2x}{2} + \frac{3\sin 4x}{2} \times \frac{1}{4} + \int \cos 2x \sin^2 2x \, dx\right.$$

$$\int \cos 2x \sin^2 2x \, dx = \int \cancel{\cos 2x}(u)^2 \frac{du}{2\cancel{\cos 2x}}$$

Let $u = \sin 2x$

$\frac{du}{dx} = 2\cos 2x$

$dx = \frac{du}{2\cos 2x}$

$$= \int u^2 \frac{du}{2}$$

$$= \frac{1}{2}\int u^2 \, du$$

$$\frac{1}{2}\left(\frac{u^3}{3} + c\right)$$

$$\frac{1}{2}\left(\frac{\sin^3 2x}{3} + c\right.$$

$$\therefore \cancel{\int \sin 6x = \frac{1}{8}\left[\frac{5}{2}x - \frac{2\sin 2x}{2} + \frac{3\sin 4x}{8}\right.}$$

$$\therefore \int \sin 6x = \frac{1}{8}\left[\frac{5}{2}x - 2\sin 2x + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6}\right] + c$$

2. $\int \cos^4 x \sin^3 x$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x = \int u^4 \cdot \cancel{\sin x} \cdot \sin^2 x \cdot \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x = -\int u^4 \cdot \sin x \, du$

$\int \cos^4 x \sin^3 x = -\int u^4 \cdot (1 - \cos^2 x) = -\int u^4 \cdot (1 - u^2)$

$\int \cos^4 x \sin^3 x = -\int u^4 - u^6 = \int u^6 - u^4$

$\int \cos^4 x \sin^3 x = \left[\frac{u^7}{7} - \frac{u^5}{5}\right]$

$\int \cos^4 x \sin^3 x = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

3. $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = du/\cos x$

$\int \cos x \sin^3 x \, dx = \int \cancel{\cos x} \; u^3 \frac{du}{\cancel{\cos x}} = \int u^3 \, du$

$\int \cos x \sin^3 x \, dx = \left[\frac{u^4}{4} + c\right]$

$\int \cos x \sin^3 x \, dx = \left[\frac{\sin^4 x}{4} + c\right]$