MBBS 100 Level

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Matric no: 19/MHS01/159

Course code: MAT 104

Instructions: Integrate the following functions:

1. $\sin^6 x$

Solution

$I = \sin^6 x$

$= (\sin^3 x)^2$

$(\sin^3 x)^2 = \left(\frac{3\sin x - \sin 3x}{4}\right)^2$

$\cancel{V = \frac{1}{4}(3\sin x - \sin 3x)^2}$

$I = \int\left(\frac{3\sin x - \sin 3x}{4}\right)^2$

$= \frac{1}{16}\int(9\sin^2 x - 6\sin x\sin 3x + \sin^2 3x)$

$= \frac{1}{16}\int\frac{9(1-\cos 2x)}{2} - 3\int 2\sin x\sin 3x + \int\frac{(1-\cos 6x)}{2}$

$= \frac{1}{16}\left[\frac{9x}{2} - \frac{\sin 2x}{4} - \frac{3\sin 2x}{2} + \frac{3\sin 4x}{4} + \frac{x}{2} - \frac{\sin 6x}{12}\right] + c$

$= \frac{1}{16}\left[5x - \frac{7\sin 2x}{4} + \frac{3\sin 4x}{4} - \frac{\sin 6x}{12}\right] + c.$

$\therefore \int\sin^6 x = \frac{1}{16}\left[5x - \frac{7\sin 2x}{4} + \frac{3\sin 4x}{4} - \frac{\sin 6x}{12}\right] + c$

2. $\int \cos^4 x \sin^3 x \, dx$

Solution:

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x \, dx = -\int u^4 \cdot \sin^3 x \cdot \frac{du}{\sin x}$

$= -\int u^4 \cdot \sin^2 x \cdot du$

$= -\int (1-\cos^2 x) \cdot u^4 \, du$

$= -\int (1-u^2) \cdot u^4 \, du$

$= -\int u^4 - u^6 \, du$

$= \int -u^4 + u^6 \, du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$\therefore \int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} + \frac{\cos^5 x}{5} + C$

$\left[\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C \right]$

3. $\int \cos x \sin^3 x$

Solution:

$u = \sin x$

$\frac{du}{dx} = \cos x$

Continuation of no. 3.

$$dx = \frac{du}{\cos x}$$

$$\int \cos x \sin^3 x = \int \cancel{\cos x} \cdot u^3 \cdot \frac{du}{\cancel{\cos x}}$$

$$= \int u^3$$

$$= \frac{u^4}{4} + C$$

$$\therefore \int \cos x \sin^3 x = \frac{\sin^4 x}{4} + C$$