Ohiomoba Britelle Ijeoma

19/MHS01/303

MBBS

2 $\int \sin^3 x \cos^4 x \, dx$

Soln

$\int \sin^3 x \cos^4 x \, dx = \int \sin^2 x \cos^4 x \sin x \, dx$

$= \int \frac{1}{2}(1 - \cos 2x) \cos^4 x \sin x \, dx$

$u = \cos x$, $du = -\sin x \, dx$; $-du = \sin x \, dx$

$2u^2 - 1 = \cos 2x$

$= -\frac{1}{2} \int (1 - (2u^2 - 1)) u^4 \, du$

$= -\frac{1}{2} \int (1 - 2u^2 + 1) u^4 \, du$

$= -\frac{1}{2} \int (2 - 2u^2) u^4 \, du$

$= -\frac{1}{2} \int 2(1 - u^2) u^4 \, du$

$= -\int (1 - u^2) u^4 \, du$

$= -\int (u^4 - u^6) \, du$

$= -\left(\frac{1}{5}\right) \cdot u^5 - \left(\frac{1}{7}\right) \cdot u^7 + c$

$= -\frac{1}{5} \cdot u^5 + \frac{1}{7} \cdot u^7 + c$

$= -\frac{1}{5}(\cos^5 x) + \frac{1}{7} \cos^7 x + c$

$= \frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x + c$

OHIOMOBA BRITELLE IDEOMA
19/MHS07/303

MBBS

$3\int \cos x \sin^3 x$

Soln

$u = \sin x$, $\frac{du}{dx} = \cos x$; $du = \cos x \, dx$

$\int \cos x \sin^3 x = \int u^3 du$

Using reverse power rule

$\int u^3 du = \frac{u^{3+1}}{3+1} + c = \frac{u^4}{4} + c$

recall, $u = \sin x$

$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + c$

OHIOMOBA BRITGLLE IJEOMA
19/MHS01/303
MBBS

1. $\int \sin^6 x$

Soln

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1 - \cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cdot \cos 2x)$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2 \cdot 2\cos^2 2x - \frac{1}{2} \cdot \cos 4x \cos 2x)$

$= \frac{1}{16}(3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2} \cdot (\cos 6x + \cos 2x))$

$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)]$

$= \frac{1}{32}(6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x)$.

So,

$I = \frac{1}{32}\int [10 - 15\cos 2x + 6\cos 4x - \cos 6x]\, dx$

$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + C$

$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + C$

$= \frac{1}{192}(60x - 45\sin 2x + 9\sin 4x - \sin 6x] + C$