NAME: ADELEKE MAHFUS ADEMOLA
MATRIC NUMBER: 19/MHS01/029
SERIAL NUMBER: 182
COURSE: MAT104

Q1. $\int \sin^6 x \, dx$

Solution

$$\int \sin^6 x \, dx = \int (\sin^2 x)^3 dx = \int \left[\frac{1-2\cos 2x}{2}\right]^3 dx$$

$$= \int \left[\frac{1-2\cos 2x}{2}\right]^3 dx$$

$$= \frac{1}{8}\int (1-2\cos 2x)^3 dx = \frac{1}{8}\int (1-2\cos 2x)^2 (1-2\cos 2x)\, dx$$

$$= \frac{1}{8}\int (1-4\cos 2x + 4\cos^2 2x)(1-2\cos 2x)\, dx$$

$$= \frac{1}{8}\int (1-4\cos 2x + 4\cos^2 2x - 2\cos 2x + 8\cos^2 2x - 8\cos^3 2x)$$

$$= \frac{1}{8}\int [1 - 6\cos 2x + 12\cos^2 2x - 8\cos^3 2x]\, dx$$

$$= \frac{1}{8}\int \left[1 - 6\cos 2x + 12\left(\frac{1+\cos 4x}{2}\right) - 8\cos^3 2x\right] dx$$

$$= \frac{1}{8}\left[\int dx - 6\int \cos 2x\, dx + 6\int (1+\cos 4x)\, dx - 8\int \cos^3 2x\, dx\right]$$

but $\int \cos^3 2x\, dx = \int \cos^2 2x \cdot \cos 2x \, dx$

$$= \int \frac{(1+\cos 4x)}{2} \cos 2x \, dx$$

$$= \frac{1}{2}\int [\cos 2x + \cos 2x \cos 4x]\, dx$$

Recall; $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$$\therefore \int \cos 2x \cos 4x = \frac{1}{2}\left[\cos 6x + \cos 2x\right]$$

$$\int \cos^3 2x\, dx = \frac{1}{2}\left[\int \cos 2x + \frac{1}{2}\left(\cos 6x + \cos 2x\right) dx\right]$$

$$= \frac{1}{2}\int \cos 2x\, dx + \frac{1}{4}\int(\cos 6x + \cos 2x)\, dx$$

$$= \frac{1}{2} \cdot \frac{\sin 2x}{2} + \frac{1}{4}\left(\frac{\sin 6x}{6} + \frac{\sin 2x}{2}\right) dx$$

$$\int \cos^3 2x\, dx = \frac{\sin 2x}{4} + \frac{1}{4}\left(\frac{\sin 6x}{6} + \frac{\sin 2x}{2}\right) dx$$

$$\int \cos^3 2x\, dx = \frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8} + C$$

$$\therefore \int (\sin^2 x)^3\, dx = \frac{1}{8}\left[\int dx - 6\int \cos 2x\, dx + 6\int(1+\cos 4x)\, dx - 8\left(\frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8}\right)\right]$$

$$= \frac{1}{8}\left[x - \cancel{6}^{3} \cdot \frac{\sin 2x}{\cancel{2}_1} + 6x + \frac{6\sin 4x}{4} - \frac{8 \cdot \sin 2x}{4} - \frac{8 \cdot \sin 6x}{24} - \frac{8\sin 2x}{8}\right]$$

$$= \frac{x}{8} - \frac{3\sin 2x}{8} + \frac{3}{4}x + \frac{3\sin 4x}{16} - \frac{\sin 2x}{4} - \frac{\sin 6x}{24} - \frac{\sin 2x}{8} + C$$

$$= \frac{7x}{8} - \frac{6\sin 2x}{8} + \frac{3}{16}\sin 4x - \frac{\sin 6x}{24} + C$$

$$\therefore \int \sin^6 x\, dx = \int (\sin^2 x)^3\, dx = \frac{7x}{8} - \frac{6\sin 2x}{8} + \frac{3}{16}\sin 4x - \frac{\sin 6x}{24} + C$$

Q $\int \cos^4 x \sin^3 x\, dx$

Solution

Since m is odd, $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$\Rightarrow dx = \frac{-du}{\sin x}$$

$$\int \cos^4 x \sin^3 x\,dx = \int \cos^4 x \sin x \cdot \sin^2 x\,dx$$
$$= \int \cos^4 x \sin x (1-\cos^2 x)\,dx$$
$$= \int u^4 \cdot \cancel{\sin x}(1-u^2) \cdot \frac{-du}{\cancel{\sin x}}$$
$$= -\int u^4(1-u^2)\,du$$
$$= -\int (u^4 - u^6)\,du$$
$$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$$
$$\therefore \int \cos^4 x \sin^3 x\,dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

Q_3 $\int \cos x \sin^3 x\,dx$

Solution

m is odd $\therefore$ $u = \cos x$

$$\frac{du}{dx} = -\sin x$$
$$\Rightarrow dx = \frac{-du}{\sin x}$$

$$\int \cos x \sin^3 x\,dx = \int \cos x \sin x \cdot \sin^2 x\,dx = \int \cos x \sin x (1-\cos^2 x)\,dx$$
$$= \int \cancel{\cos x} \cdot \cancel{\sin x}(1-u^2) \cdot \frac{-du}{\cancel{\sin x}}$$
$$= -\int u(1-u^2)\,du = -\int (u-u^3)\,du$$
$$= -\left[\frac{u^2}{2} - \frac{u^4}{4}\right] + C$$
$$= \frac{u^4}{4} - \frac{u^2}{2} + C$$
$$\therefore \int \cos x \sin^3 x\,dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C$$