NAME : AMADI SARIMA MILDRED
DEPARTMENT : MEDICINE AND SURGERY
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COURSE : MAT 104

1. $\sin^6 x$

SOLUTION :

NOTE :
- $2\sin^2\theta = 1 - \cos 2\theta$

$$I = \int \sin^6 x \, dx$$

$$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$$

$$\sin^6 x = \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$\sin^6 x = \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$$

$$\sin^6 x = \frac{1}{16}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$$

$$\sin^6 x = \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$$

$$\sin^6 x = \frac{1}{16}\left[3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x\right]$$

$$\sin^6 x = \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \frac{1}{2}\cdot 2\cos 4x \cos 2x\right]$$

$$\sin^6 x = \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$\sin^6 x = \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$\sin^6 x = \frac{1}{32}\left[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right]$$

$$\sin^6 x = \frac{1}{32}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]$$

$$I = \frac{1}{32}\int \left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right] dx$$

$$I = \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + C$$

$$I = \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + C$$

$$I = \frac{1}{192}\left[60x - 45\sin 2x + 9\sin 4x - \sin 6x\right] + C$$

2. $\cos^4 x \sin^3 x$

$$\int \cos^4 x \sin^3 x = \int \cos^4 x \sin^3 x \, dx$$

$$\cos^4 x \sin^3 x = \cos^4 x \sin(x) \cdot \sin^2(x)$$

$$\cos^4 x \sin^3 x = \cos^4 x \sin x \left[1 - \cos^2(x)\right]$$

Let $u = \sin x$

$$\frac{du}{dx} = -\cos(x)$$

$$-du = \cos x \, dx$$

$$\cos^4 x \sin^3 x = u^4 \sin x \left[1 - u^2\right]$$

$$\int = \int u^4 \sin x \left[1 - u^2\right] dx$$

$$\int \sin(x) \, dx \, (1 - u^2) u^4$$

$$\int -du \, (1 - u^2) u^4$$

$$\int (u^4 - u^6) \, du = \frac{u^5}{5} - \frac{u^7}{7} + C$$

$$= -\frac{\cos^5(x)}{5} + \frac{\cos^7(x)}{7}$$

$$\int \sin^m(x) \cos^n(x) \, dx$$

$$\int \cos^n(x) \cdot \sin(x) \cdot \sin^{(m-1)} \, dx$$

$$\therefore \int \cos^4 x \sin^3 x \, dx = -\left(\frac{-1}{7} \cos^7(x) + \frac{1}{5} \cos^5(x)\right) + C$$

3. $\cos x \sin^3 x$

SOLUTION :

$$\int \cos x \sin^3 x \, dx$$

$u = \cos x$

$du = \sin(x) \, dx$

$$\int \cos x \sin^3 x = u^3 \, du$$

$$\frac{u^3}{du} = \frac{u^4}{4}$$

$$\frac{u^4}{4} = \frac{\cos^4(x)}{4} + C$$

$$\therefore \int \cos x \sin^3 x \, dx = \frac{\cos^4(x) + C}{4}$$