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MATRIC NUMBER: 19/MH001/121

1) $\int \sin^6 x$

$= \int \sin^2 x \left(\sin^2 x\right)^2$

$= \int \sin^2 x \left(1 - \cos^2 x\right)^2$

$\int \sin^2 x \left(1 - 2\cos^2 x + \cos^4 x\right)$

$\int \sin^2 x - 2\sin^2 x \cos^2 x + \sin^2 x \cos^4 x$

$\int 2\sin^2 x \cos^2 x = 2\int \sin^2 x \cos^2 x$

$= 2\int \sin^2 x \left(1 - \sin^2 x\right)$

$= 2\int \sin^2 x - \sin^4 x$

where $\int \sin^4 x = \int (\sin^2 x)^2$

$$= \int \left(\frac{1 - \cos 2x}{2}\right)^2$$

$$= \frac{1}{4}\int (1 - \cos 2x)^2$$

$$= \frac{1}{4}\int 1 - 2\cos 2x + \cos^2 2x$$

$$= \frac{1}{4}\int 1 - 2\cos 2x + \frac{1}{2} + \frac{\cos 4x}{2}$$

$$= \frac{1}{4}\left[x - \frac{2\sin 2x}{2} + \frac{x}{2} + \frac{\sin 4x}{8}\right] + c$$

$$= \frac{1}{4}\left[\frac{3x}{2} - \frac{2\sin 2x}{2} + \frac{\sin 4x}{8}\right] + c$$

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$$= \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c$$

$$2\int \sin^2 x - \sin^4 x = 2\left[\frac{x}{2} - \frac{\sin 2x}{4} - \frac{3x}{8} + \frac{\sin 2x}{4} - \frac{\sin 4x}{32}\right] + c$$

$$= \left[x - \frac{\sin 2x}{2} - \frac{3x}{4} + \frac{\sin 2x}{2} - \frac{\sin 4x}{16}\right] + c$$

$$\int 2\sin^2 x \cos^2 x = \left[\frac{x}{4} - \frac{\sin 4x}{16}\right] + c$$

$$\int \sin^2 x \cos^4 x = \int \sin^2 x (\cos^2 x)^2$$

$$\int \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)^2$$

$$\frac{1}{8}\int (1 - \cos 2x)(1 + \cos 2x)^2$$

$$= \frac{1}{8}\int (1 - \cos 2x)(1 + 2\cos 2x + \cos^2 2x)$$

$$= \frac{1}{8}\int (1 + 2\cos 2x + \cos^2 2x - \cos 2x - 2\cos^2 2x - \cos^3 2x)$$

$$= \frac{1}{8}\int 1 + \cos 2x - \cos^2 2x - \cos^3 2x$$

$$= \frac{1}{8}\int 1 + \cos 2x - \left(\frac{1 + \cos 4x}{2}\right) - (\cos 2x (\cos^2 2x))$$

$$= \frac{1}{8}\int 1 + \cos 2x - \frac{1}{2} - \frac{\cos 4x}{2} - (\cos 2x (1 - \sin^2 2x))$$

$$= \frac{1}{8}\int 1 + \cos 2x - \frac{1}{2} - \frac{\cos 4x}{2} - (\cos 2x - \sin^2 2x \cos 2x)$$

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$$= \frac{1}{8}\int 1 + \cos 2x - \frac{1}{2} - \frac{\cos 4x}{2} - \cos 2x + \sin^2 2x \cos 2x$$

$$= \frac{1}{8}\int \frac{1}{2} - \frac{\cos 4x}{2} + \sin^2 2x \cos 2x$$

$$\int \sin^2 2x \cos 2x \, dx$$

let $u = \sin 2x$

$\frac{du}{dx} = 2\cos 2x$, $dx = \frac{du}{2\cos 2x}$

$$\int \sin^2 2x \cos 2x \, dx = \int u^2 \cos 2x \times \frac{du}{2\cos 2x}$$

$$= \int u^2 \times \frac{du}{2}$$

$$= \frac{1}{2}\int u^2 du$$

$$= \frac{1}{2}\left[\frac{u^3}{3}\right]$$

$$= \frac{1}{2}\left[\frac{\sin^3 2x}{3}\right] + c$$

$$\therefore \int \sin^2 2x \cos 2x \, dx = \frac{\sin^3 2x}{6} + c$$

$$\frac{1}{8}\int \frac{1}{2} - \frac{\cos 4x}{2} + \sin^2 2x \cos 2x$$

$$\frac{1}{8}\left[\frac{x}{2} - \frac{\sin 4x}{8} + \frac{\sin^3 2x}{48}\right] + c$$

$$\int \sin^2 x \cos^4 x = \left[\frac{x}{16} - \frac{\sin 4x}{64} + \frac{\sin^3 2x}{48}\right] + c$$

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$$\int \sin^2 x - 2\sin^2 x\cos^2 x + \sin^2 x\cos^4 x$$

$$= \left[\frac{x}{2} - \frac{\sin 2x}{4} - \frac{x}{4} + \frac{\sin 4x}{16} + \frac{x}{16} - \frac{\sin 4x}{64} + \frac{\sin^3 2x}{48}\right]$$

$$= \left[\frac{5x}{16} - \frac{\sin 2x}{4} + \frac{3\sin 4x}{64} + \frac{\sin^3 2x}{48}\right] + c$$

$$\therefore \int \sin^6 x = \left[\frac{5x}{16} - \frac{\sin 2x}{4} + \frac{3\sin 4x}{64} + \frac{\sin^3 2x}{48}\right] + c$$

2) $\int \cos^4 x \sin^3 x$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$, $dx = \frac{-du}{\sin x}$

$$\int \cos^4 x \sin^3 x\, dx = \int u^4 \cancel{\sin^3 x} \times \frac{-du}{\cancel{\sin x}}$$

$$= -\int u^4 \times \sin^2 x\, du$$

$$= -\int u^4 \times (1 - \cos^2 x)\, du$$

$$= -\int (1-u^2)u^4\, du$$

$$= -\int (u^4 - u^6)\, du$$

$$= \int (u^6 - u^4)\, du$$

$$= \left[\frac{u^7}{7} - \frac{u^5}{5}\right]$$

$$\int \cos^4 x \sin^3 x\, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

5) $\int \cos x \sin 3x \, dx$

$$\int \cos x \sin 3x \, dx = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\int \cos x \sin 3x \, dx = \frac{1}{2}\int \left[\sin 4x - \sin(-2x)\right]$$

$$\int \cos x \sin 3x \, dx = \frac{1}{2}\int \left[\sin 4x + \sin 2x\right]$$

$$= \frac{1}{2}\left[-\frac{1}{4}\cos 4x - \frac{1}{2}\cos 2x\right]$$

$$= \frac{-\cos 4x}{8} - \frac{\cos 2x}{4} + c$$

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