

$$1. \int \sin^6 x \, dx$$

$$= \int (\sin^2 x)^3 \, dx$$

Recall that  $\sin^2 x = 1 - \cos^2 x$

$$\therefore \int (\sin^2 x)^3 \, dx = \int (1 - \cos^2 x)^3 \, dx$$

$$= \int (1 - \cos^2 x)(1 - \cos^2 x)(1 - \cos^2 x) \, dx$$

$$= \int (1 - \cos^2 x)(1 - 2\cos^2 x + \cos^4 x) \, dx$$

$$= \int (1 - 3\cos^2 x + \cos^4 x - \cos^6 x) \, dx$$

$$= \int 1 - 3 \int \cos^2 x + \int \cos^4 x - \int \cos^6 x$$

$$= x - 3 \left( \frac{\cos^3 x}{3} \right) + \frac{\cos^5 x}{5} - \frac{\cos^7 x}{7}$$

$$= x - \cos^3 x + \frac{\cos^5 x}{5} - \frac{\cos^7 x}{7}$$

$$2. \int \cos^4 x \sin^3 x \, dx$$

Let  $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{-du}{\sin x}$$

$$\therefore \int \cos^4 x \sin^3 x \, dx$$

$$= \int u^4 \frac{\sin^2 x}{\sin x} \cdot \frac{-du}{\sin x}$$

$$= \int u^4 \sin^2 x \cdot -du$$

$$= \int u^4 (1 - \cos^2 x) - du$$

$$= - \int u^4 (1 - u^2) \, du$$

$$= - \int u^4 - u^6 \, du$$

$$= - \left( \frac{u^5}{5} - \frac{u^7}{7} \right)$$

$$= - \frac{\cos^5 x}{5} + \frac{\cos^7 x}{7}$$

$$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5}$$

$$3. \int \cos x \sin^3 x dx$$

$$\text{Let } u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$dx = -\frac{du}{\sin x}$$

$$\therefore \int \cos x \sin^3 x dx$$

$$= \int u \cdot \sin^3 x \cdot \frac{-du}{\sin x}$$

$$= - \int u \cdot \sin^2 x du$$

$$= - \int u \cdot (1 - \cos^2 x) du$$

$$= - \int u \cdot (1 - u^2) du$$

$$= - \int u - u^3 du$$

$$= - \left( \frac{u^2}{2} - \frac{u^4}{4} \right)$$

$$= -\frac{u^2}{2} + \frac{u^4}{4}$$

$$= -\frac{\cos^2 x}{2} + \frac{\cos^4 x}{4}$$

$$= \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2}$$