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DEPARTMENT :- MBBS

MATRIC NO:- 19/MHS01/277

COURSE:- MAT 104

ASSIGNMENT

Integrate the following functions

1.) $\sin^6 x$

Solution

$\int \sin^6 x$

$\int \sin^2 x(\sin^2 x)^2$

$\int \sin^2 x(-\cos^2 x)^2$

$\int \sin^2 x(1-2\cos^2 x+\cos^4 x)$

$\int \sin^2 x - 2\sin^2 x\cos^2 x + \sin^2 x\cos^4 x$

$\int 2\sin^2 x\cos^2 x = 2\int \sin^2 x\cos^2 x$

$= 2\int \sin^2 x(-\sin^2 x)$

$= 2\int \sin^2 x - \sin^4 x$

$\int \sin^4 x$

$\int (\sin^2 x)^2$

$$\int \left(\frac{1-\cos 2x}{2}\right)^2$$

$$\frac{1}{4}\int (1-\cos 2x)^2$$

$$\frac{1}{4}\int 1-2\cos 2x+\cos^2 2x$$

$$\frac{1}{4}\int 1-2\cos 2x+\frac{1}{2}+\frac{\cos 4x}{2}$$

$$\frac{1}{4}\left[x-\frac{2\sin 2x}{2}+\frac{x}{2}+\frac{\sin 4x}{8}\right]+C$$

$$\frac{1}{4}\left[\frac{3x}{2}-\frac{\cancel{2}\sin 2x}{\cancel{2}}+\frac{\sin 4x}{8}\right]+C$$

$$\frac{3x}{8}-\frac{\sin 2x}{4}+\frac{\sin 4x}{32}+C$$

$$2\int \sin^2 x-\sin^4 x = 2\left[\frac{x}{2}-\frac{\sin 2x}{4}-\frac{3x}{8}+\frac{\sin 2x}{4}-\frac{\sin 4x}{32}\right]+C$$

$$=\left[x-\frac{\sin 2x}{2}-\frac{3x}{4}+\frac{\sin 2x}{2}-\frac{\sin 4x}{16}\right]+C$$

$$\int \cancel{2}\sin^2 x\cos^2 x=\left[\frac{x}{4}-\frac{\sin 4x}{16}\right]+C$$

$$\int \sin^2 x\cos^4 x$$

$$\int \sin^2 x\cos^4 x$$

$$\int \sin^2 x(\cos^2 x)^2$$

$$\int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2$$

$$\frac{1}{8}\int (1-\cos 2x)(1+\cos 2x)^2$$

$$\frac{1}{8}\int (1-\cos 2x)(1+2\cos 2x+\cos^2 2x)$$

$\frac{1}{8}\int 1 + 2\cos 2x + \cos^2 2x - \cos 2x - 2\cos^2 2x - \cos^3 2x$

$\frac{1}{8}\int 1 + \cos 2x + \cos^2 2x - \cos^3 2x$

$\frac{1}{8}\int 1 + \cos 2x - \left(\frac{1+\cos 4x}{2}\right) - \left(\cos 2x(\cos^2 2x)\right)$

$\frac{1}{8}\int 1 + \cos 2x - \frac{1}{2} - \frac{\cos 4x}{2} - \left(\cos 2x(1-\sin^2 2x)\right)$

$\frac{1}{8}\int 1 + \cos 2x - \frac{1}{2} - \frac{\cos 4x}{2} - \left(\cos 2x - \sin^2 2x \cos 2x\right)$

$\frac{1}{8}\int 1 + \cos 2x - \frac{1}{2} - \frac{\cos 4x}{2} - \cos 2x + \sin^2 2x \cos 2x$

$\frac{1}{8}\int \frac{1}{2} - \frac{\cos 4x}{2} + \sin^2 2x \cos 2x$

$\int \sin^2 2x \cos 2x \, \delta x$

Let $u = \sin 2x$

$\frac{\delta u}{\delta x} = 2\cos 2x$, $\delta x = \frac{\delta u}{2\cos 2x}$

$\int \sin^2 2x \cos 2x \, \delta x = \int u^2 . \cos 2x \cdot \frac{\delta u}{2\cos 2x}$

$= \int u^2 . \frac{\delta u}{2}$

$= \frac{1}{2}\int u^2 \delta u$

$= \frac{1}{2}\left[\frac{u^3}{3}\right] + C$

$= \frac{1}{2}\left[\frac{\sin^3 2x}{3}\right] + C$

$$\int \sin^2 2x \cos 2x \, \delta x = \frac{\sin^3 2x}{6} + C$$

$$\frac{1}{8} \int \frac{1}{2} - \frac{\cos 4x}{2} + \sin^2 2x \cos 2x$$

$$\frac{1}{8} \left[\frac{x}{2} - \frac{\sin 4x}{8} + \frac{\sin^3 2x}{6} \right] + C$$

$$\left[\frac{x}{16} - \frac{\sin 4x}{64} + \frac{\sin^3 2x}{48} \right] + C$$

$$\int \sin^2 x \cos^4 x = \left[\frac{x}{16} - \frac{\sin 4x}{64} + \frac{\sin^3 2x}{48} \right] + C$$

$$\int \sin^2 x - 2 \sin^2 x \cos^2 x + \sin^2 x \cos^4 x$$

$$\cancel{\left[\frac{x}{2} - \frac{\sin 2x}{4} - \frac{x}{4} + \frac{\sin 4x}{16} + \frac{x}{16} - \frac{3\sin 4x}{16} \right]}$$

$$\left[\frac{x}{2} - \frac{\sin 2x}{4} - \frac{x}{4} + \frac{\sin 4x}{16} + \frac{x}{16} - \frac{\sin 4x}{64} + \frac{\sin^3 2x}{48} \right] + C$$

$$\left[\frac{5x}{16} - \frac{\sin 2x}{4} + \frac{3 \sin 4x}{64} + \frac{\sin^3 2x}{48} \right] + C$$

$$\int \sin^6 x = \left[\frac{5x}{16} - \frac{\sin 2x}{4} + \frac{3 \sin 48}{64} + \frac{\sin^3 2x}{48} \right] + C$$

② $\cos^4 x \sin^3 x$

Solution

$\int \cos^4 x \sin^3 x \, \delta x$

Let $u = \cos x$

$\frac{\delta u}{\delta x} = \sin x, \; \delta x = -\frac{\delta u}{\sin x}$

$$\int \cos^4 x \sin^3 x \, \delta x = \int u^4 \cdot \sin^3 x \cdot \frac{-\delta u}{\cancel{\sin x}}$$

$$= -\int u^4 \cdot \sin^2 x \, \delta u$$

$$= -\int (1 - \cos^2 x) \cdot u^4 \, \delta u$$

$$= -\int (1 - u^2) u^4 \, \delta u$$

$$= \int (u^2 - 1) u^4 \, \delta u$$

$$= \int (u^6 - u^4) \, \delta u$$

$$= \left[\frac{u^7}{7} - \frac{u^5}{5} \right] + c$$

$$\int \cos^4 x \sin^3 x = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

(3) $\cos x \sin^3 x$

Solution

$$\int \cos x \sin^3 x$$

Let $u = \sin x$

$$\frac{\delta u}{\delta x} = \cos x, \quad \delta x = \frac{\delta u}{\cos x}$$

$$\int \cos x \sin^3 x = \int \cancel{\cos x} \cdot u^3 \frac{\delta u}{\cancel{\cos x}}$$

$$\int \cos x \sin^3 x = \left[\frac{u^4}{4}\right] + c$$

$$\int \cos x \sin^3 x = \frac{\sin^4 x}{4} + c$$