

$$i) \int \cos x \sin 3x \, dx$$

$$\sin 3x = 3 \cos^2 x \sin x - \sin^3 x$$

$$\sin^2(x) = 1 - \cos^2 x$$
$$= \int -(\cos x - 4 \cos^3 x) \sin x \, dx$$

$$\cos x = u$$

$$-\sin x \, dx = du$$

$$= -\int (4u^3 - u) du$$

$$= \int (u - 4u^3) du$$

$$= \frac{u^2}{2} - u^4 + C$$

$$= \frac{\cos^2 x}{2} - \cos^4 x + C$$

$$1) \int \sin^6 x dx$$

$$\int \sin^6 x dx$$

using reduction formula

$$\int \sin^n x dx = \frac{n-1}{n} \int \sin^{n-2} x dx - \frac{\cos x \sin^{n-1} x}{n}$$

$$\int \sin^6 x dx = -\frac{\cos x \sin^5 x}{6} + \frac{5}{6} \int \sin^4 x dx$$

$$\int \sin^4 x dx = -\frac{\cos x \sin^3 x}{2} + \frac{3}{4} \int \sin^2 x dx$$

$$\int \sin^2 x dx = -\frac{\cos x \sin x}{2} + \frac{1}{2} \int dx$$

$$= -\frac{\cos x \sin x}{2} + \frac{1}{2} x$$

$$\int \sin^4 x dx = \frac{\cos x \sin^3 x}{4} - \frac{3}{8} \cos x \sin x + \frac{3x}{8}$$

$$\int \sin^6 x dx = \frac{\cos x \sin^5 x}{6} - \frac{5}{24} \cos x \sin^3 x$$

$$-\frac{5}{16} \cos x \sin x + \frac{5x}{16} + C$$

$$2) \int \cos^4 x \sin^3 x dx$$

$$\int \cos^4 x (\sin^2 x) \sin x dx$$

$$\int \cos^4 x (1 - \cos^2 x) \sin x dx$$

$$\cos x = t$$

$$\sin x dx = -dt$$

$$= - \int t^4 (1 - t^2) dt$$

$$= \int (t^6 - t^4) dt$$

$$= \frac{t^7}{7} - \frac{t^5}{5} + C$$

$$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$