Math 104

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(1) $\int \sin^6 x = \int \sin^2 x \cdot (\sin^2 x)^2$

$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)^2$

$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-2\cos 2x+\cos^2 2x}{4}\right)$

$= \frac{1}{8}\int 1-3\cos 2x+3\cos^2 2x - \cos^3 2x\, dx$

$= \frac{1}{8}\int 1-3\cos 2x+3\left(\frac{1+\cos 4x}{2}\right) - \cos^3 2x\, dx$

$\int \cos^3 2x\, dx = \cos 2x \cdot \cos^2 2x\, dx$

$= \cos 2x\,(1-\sin^2 2x)\, dx$

$u = \sin 2x$

$\frac{du}{dx} = 2\cos 2x \qquad dx = \frac{du}{2\cos 2x}$

$= \int \cos 2x - \cos 2x \sin^2 2x \times \frac{du}{2\cos 2x}$

$= \frac{1}{2}\int \cos 2x - \int \sin^2 2x \cdot du$

$= \frac{1}{2}\int \cos 2x - \int u^2\, du$

$= \frac{\sin 2x}{4} - \frac{\sin^3 2x}{6} + C$

$$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left(\frac{1+\cos 4x}{2}\right) - \frac{1}{2}\left[\cos 2x - u^2 du\right]$$

$$= \frac{1}{8}\left[\frac{5x}{2} - \frac{3\sin 2x}{2} + \frac{3\sin 4x}{8} - \frac{\sin 2x}{4} + \frac{\sin^3 2x}{6} + C\right.$$

$$\boxed{= \frac{5x}{16} - \frac{3\sin 2x}{16} + \frac{3\sin 4x}{64} - \frac{\sin 2x}{32} + \frac{\sin^3 2x}{48} + C}$$

(2) $\int \cos^4 x \sin^3 x$

$u = \cos x$

$= \quad \frac{du}{dx} = -\sin x \quad dx = \frac{-du}{\sin x}$

$\int u^4 \cdot \cos$

$\int \cos^4 x \cdot \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$

$\int \cos^4 x \cdot \sin^2 x \cdot -du$

$= \int u^4 (1 - \cos^2 x) - du$

$= \int u^4 (1 - u^2) - du$

$= \int (u^4 - u^6) - du$

$= \int (u^6 - u^4) du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$\boxed{= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C}$

(3) $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$= \int \cos x \cdot \sin x \cdot \sin^2 x \, dx$

$= \int \cos x \cdot \sin x \cdot \sin^2 x \cdot \frac{du}{\cos x}$

$= \int \sin x \cdot \sin^2 x \cdot du$

$= \int u \, (u^2) \cdot du$

$= \int u^3 \, du$

$= \frac{u^4}{4} + C$

$\boxed{= \frac{\sin^4 x}{4} + C}$