MAT ASSIGNMENT

1 $\int \sin^6 x$

$= \int (\sin^2 x)^3$

$\sin^2 x = \frac{1}{2}[1 - \cos 2x]$

$\sin^6 x = \left[\frac{1-\cos 2x}{2}\right]^3$

$\sin^6 x = \frac{1}{8}\int (1-\cos 2x)^3$

$\sin^6 x = \frac{1}{8}\int \cancel{\cos 2x} = \frac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$

$= \frac{1}{8}\int (1 - \cos 2x - 2\cos 2x + 2\cos^2 2x + \cos^2 2x - \cos^3 2x)$

$= \frac{1}{8}\int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$

$= \frac{1}{8}\int 1 - 3\cos 2x + 3\left(\frac{1+\cos 4x}{2}\right) - \cos 2x(1 - \sin^2 2x)$

$= \frac{1}{8}\int \frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x \, dx$

$= \frac{1}{8}\left[\frac{5x}{2} - 2\sin 2x + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6}\right] + c$

$= \frac{x}{2} - \frac{\sin 2x}{4} + \frac{3\sin 4x}{64} + \frac{\sin^3 2x}{48} + C$

2 $\int \cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx \,, \; dx = \frac{du}{-\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\int \cancel{\sin x} \cdot \sin^2 x \cdot u^4 \frac{-du}{\cancel{\sin x}}$

$\int \cos^4 x \sin^3 x = \cancel{\int \sin x (1} - \int (1 - \cos^2 x) u^4 \, du$

$\int \cos^4 x \sin^3 x = -\int (1 - u^2) u^4 \, du$

$\int \cos^4 x \sin^3 x = -\int (u^4 - u^6) \, du$

$\int \cos^4 x \sin^3 x = -\left[\frac{u^5}{5} - \frac{u^7}{7} \right] + C$

$\int \cos^4 x \sin^3 x = +\frac{u^7}{7} - \frac{u^5}{5} + C = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3 $\int \cos x \sin 3x = \frac{1}{2} \int \sin(x+3x) - \sin(x - 3x)$

$\int \cos x \sin^3 x = \frac{1}{2} \int \sin 4x + \sin 2x$

$\int \cos x \sin^3 x = \frac{1}{2}\left[-\frac{\cos 4x}{4} - \frac{\cos 2x}{2}\right] + C$

$\int \cos x \sin^3 x = -\frac{\cos 4x}{8} - \frac{\cos 2x}{4} + C$