

1) $\int \sin^6 x \, dx$

Using reduction formula

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \int \sin^4 x \, dx$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left[-\frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \int \sin^2 x \, dx \right]$$

$$= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x + \frac{5}{8} \left[-\frac{1}{2} \sin x \cos x + \frac{1}{2} \int \sin^0 x \, dx \right]$$

$$= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x - \frac{5}{16} \sin x \cos x + \frac{5}{16} x + C$$

2) $\int \cos^4 x \sin^3 x \, dx$

$$\int \sin x \times \sin^2 x (\cos^2 x)^2 \, dx$$

$$\int \sin x [1 - \cos^2 x] (\cos^2 x)^2 \, dx$$

$$u = \cos x \quad du = -\sin x \, dx = \frac{du}{-\sin x}$$

$$= -\int \sin x [1 - \cos^2 x] (\cos^2 x)^2 \frac{du}{\sin x}$$

$$= -\int [1 - u^2] [u^2]^2 \, du$$

$$= -\int [1 - u^2] [u^4] \, du$$

$$= -\int [u^4 - u^6] \, du$$

$$= -\frac{u^5}{5} + \frac{u^7}{7} + C$$

$$= -\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x + C$$

3) $\int \cos x \sin^3 x \, dx$

$$\int \cos x \sin x \sin^2 x \, dx$$

$$\int \sin x (1 - \cos^2 x) \cos x \, dx$$

$$u = \cos x \quad du = -\sin x \, dx = \frac{du}{-\sin x}$$

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$$-\int \cancel{\sin x} [1-u^2] u \cdot \frac{du}{\cancel{\sin x}}$$

$$-\int [1-u^2] u \cdot du$$

$$-\int [u - u^3] du$$

$$-\frac{u^2}{2} + \frac{u^4}{4} + C$$

$$-\frac{1}{2} \cos^2 x + \frac{1}{4} \cos^4 x + C$$