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Assignment

1. $\sin^6 x$
2. $\cos^4 x \sin^3 x$
3. $\cos^2 x \sin^3 x$

solution

1. $\int \sin^6 x \Rightarrow \int \sin^6 x \, dx$

$$\int \sin^6 x = \int (\sin^2 x)^2 (\sin^2 x)$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$$

$$= \frac{1}{8}\int \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1 - \cos 2x)$$

$$= \frac{1}{16}\int (2 - 4\cos 2x + 1 + \cos 4x)(1 - \cos 2x)$$

$$= \frac{1}{16}\int (3 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$$

$$= \frac{1}{16}\int (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$$

$$= \frac{1}{16}\int (3 - 7\cos 2x + \cos 4x + (2(2\cos^2 2x) - \tfrac{1}{2} \cdot 2\cos 4x \cos 2x)$$

$$= \frac{1}{16}\int (3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)$$

$$= \frac{1}{16}\int \left(3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right)$$

$$= \frac{1}{320}\int (6 - 16\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x)$$

$= \frac{1}{32}\int [10 - 15\cos 2x + 6\cos 4x - \cos 6x]\,dx$

$\int \sin^6 x = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + 6\frac{\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$

$= \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$

2. $\int \cos^4 x \sin^3 x\,dx = \int \sin x \cdot \sin^2 x \cdot \cos^4 x\,dx$

$= \int \sin x \cdot (1 - \cos^2 x) \cdot \cos^4 x\,dx$

Let $v = \cos x$

$\frac{dv}{dx} = -\sin x \therefore dx = \frac{-dv}{\sin x}$

$= \int \cancel{\sin x} \cdot (1 - \cos^2 x) \cdot v^4 \cdot \frac{-dv}{\cancel{\sin x}}$

$= -\int (1 - \cos^2 x) \cdot v^4 \cdot dv$

$= -\int (1 - v^2) \cdot v^4 \cdot dv$

$= -\int (v^4 - v^6) \cdot dv$

$= -\left[\frac{v^5}{5} - \frac{v^7}{7}\right] + C$

$= \frac{v^7}{7} - \frac{v^5}{5} + C$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

$= -\frac{\cos 4x}{8} - \frac{\cos 2x}{4} + C$

3. $\cos x \sin^3 x \Rightarrow \int \cos x \sin^3 x \, dx$

$u = \sin x$

$du/dx = \cos x \therefore dx = du/\cos x$

$= \int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$

$= \int u^3 \, du = \frac{u^4}{4} + C$

$= \frac{\sin^4 x}{4} + C_{11}$

$x^2 x^4 = x^6 = y$