

Name: Ufomaduh Kelly Ugonna
Course code: MAT104
Matric. No: 19/MHS01/413

1) $\int \sin^6 x \, dx$

Sol

$$\begin{aligned}\int \sin^6 x \, dx &= \int (\sin^2 x)^3 \, dx \\ &= \int \left(\frac{1 - \cos 2x}{2} \right)^3 \, dx \\ &= \frac{1}{8} \int (1 - \cos 2x)^3 \, dx \\ &= \frac{1}{8} \int (1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x) \, dx \\ &= \frac{1}{8} \left(x - \frac{3\sin 2x}{2} + \left(3 \times \frac{x}{2} + \frac{\sin 4x}{8} \right) - \frac{\sin 2x \cos^2 2x}{2} \right) \\ &= \frac{1}{8} \left(\frac{5x}{2} - \frac{3\sin 2x}{2} + \frac{3\sin 4x}{8} - \frac{\sin 2x \cos^2 2x}{2} \right) + C \\ &= \frac{1}{8} \left(\frac{5x}{2} - 2\sin 2x + \frac{3\sin^4 x}{8} + \frac{\sin^2 2x}{6} \right) + C\end{aligned}$$

$$2) \int \cos^4 x \sin^3 x \, dx$$

Sol

$$\int \cos^4 x \sin^3 x \, dx$$

$$u = \cos x$$

$$du = -\sin x \, dx$$

$$dx = \frac{du}{-\sin x}$$

$$dx = -\frac{du}{\sin x}$$

$$\int u^4 \sin x \cdot \sin^2 x \cdot \frac{du}{-\sin x}$$

$$= -\int u^4 \sin^2 x \, du$$

$$= -\int (1 - \cos^2 x) u^4 \, du$$

$$= -\int (1 - u^2) u^4 \, du$$

$$= -\int u^4 - u^6 \, du$$

$$= -\left(\frac{u^5}{5} - \frac{u^7}{7} \right) + C$$

$$\int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

$$3) \cos x \sin^3 x$$

Sol

$$\int \cos x \sin^3 x \, dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$dx = \frac{du}{\cos x}$$

$$\int \cos x \cdot u^3 \frac{du}{\cos x}$$

$$\int u^3 \, du$$

$$= \frac{u^4}{4} + c$$

$$\int \cos x \sin^3 x = \frac{\sin^4 x}{4} + c.$$