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Course: Mat 104

1. $\int \sin^6 x \, dx$

Solution.

$$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$$
$$\sin^6 x = \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \cos^2 2x\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{18}\left(3 - 4\cos 2x + \cos 4x\right)\left(1 - \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 3\cos 2x - 4\cos 2x + 4\cos^2 2x + \cos 4x - \cos 4x \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 7\cos 2x + 2 \times 2\cos^2 x + \cos 4x - \frac{1}{2} \cdot 2\cos 4x \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right)$$

$$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right)$$

$$= \frac{1}{32}\left(6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x + \cos 2x\right)$$

$$= \frac{1}{32}\left(10 - 15\cos 2x + 6\cos 4x - \cos 6x\right)$$

$$\int \sin^6 x \, dx = \frac{1}{32}\int\left(10 - 15\cos 2x + 6\cos 4x - \cos 6x\right)$$
$$\int \sin^6 x \, dx = \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + C$$

$$\therefore \int \sin^6 x \, dx = \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + C$$

2. $\int \cos^4 x \sin^3 x \, dx$

Solution

$u = \cos x \qquad dx = \frac{-du}{\sin x}$

$\int \cos^4 x \sin^3 x \, dx$

$= \int u^4 . \sin x (\sin^2 x) dx$

$= \int u^4 \cdot \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$

$= -\int u^4 \cdot \sin^2 x \cdot du$

$= -\int u^4 \cdot (1 - \cos^2 x) \cdot du$

$= -\int u^4 \cdot (1 - u^2) \cdot du$

$= -\int u^4 - u^6 \, du$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + c$

$= \frac{u^7}{7} - \frac{u^5}{5} + c$

$\therefore \int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

3. $\int \cos x \sin^3 x \, dx$

$u = \sin x \qquad dx = \frac{du}{\cos x}$

$\int \cos x \sin^3 x \, dx$

$= \int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$

$= \int u^3 du = \frac{u^4}{4} + c$

$\therefore \int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + c$