

$$1 \int \sin^6 x \, dx$$

Solution

$$\int \sin^6 x = \int \sin^2 x \cdot \sin^4 x \, dx$$

$$\int \sin^6 x = \int \frac{1 - \cos 2x}{2} \cdot \left(\frac{1 - \cos 2x}{2} \right)^2 dx$$

$$\int \sin^6 x = \int \frac{1 - \cos 2x}{2} \cdot \frac{1 - 2\cos 2x + \cos^2 2x}{4} dx$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 2\cos 2x + \cos^2 2x - \cos 2x + 2\cos^2 2x - \cos^3 2x \, dx$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + 3 \left(\frac{1 + \cos 4x}{2} \right) - \cos 2x (1 - \sin^2 x) \, dx$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + \frac{3}{2} + \frac{3\cos 4x}{2} - \cos 2x + \cos 2x \sin^2 x \, dx$$

$$\int \sin^6 x = \frac{1}{8} \int \left[\frac{5}{2} - 4 \cos 2x + \frac{3 \cos 4x}{2} + \cos 2x \sin^2 2x \right] dx$$

~~$$\int \sin^6 x = \frac{1}{8} \left[\frac{5x}{2} - \frac{4 \sin 2x}{2} + \frac{3 \sin 4x}{2} \right] + \int \cos 2x \sin^2 2x dx$$~~

$$\int \sin^6 x = \frac{1}{8} \left[\frac{5x}{2} - \frac{4 \sin 2x}{2} + \frac{3 \sin 4x}{2} \times \frac{1}{4} + \int \cos 2x \sin^2 2x dx \right]$$

$$\int \cos 2x \sin^2 2x dx = \int \cos 2x (u)^2 \frac{du}{2 \cos 2x}$$

$$\left. \begin{array}{l} \text{let } u = \sin 2x \\ \frac{du}{dx} = 2 \cos 2x \\ dx = \frac{du}{2 \cos 2x} \end{array} \right\} = \int \frac{u^2 du}{2} = \frac{1}{2} \int u^2 du = \frac{1}{2} \left(\frac{u^3}{3} + C \right) = \frac{1}{2} \left(\frac{\sin^3 2x}{3} + C \right)$$

~~$$\therefore \int \sin^6 x = \frac{1}{8} \left[\frac{5x}{2} - \frac{2 \sin 2x}{2} + \frac{3 \sin 4x}{8} \right]$$~~

$$\therefore \int \sin^6 x = \frac{1}{8} \left[\frac{5x}{2} - 2 \sin 2x + \frac{3 \sin 4x}{8} + \frac{\sin^3 2x}{6} \right] + C$$

2. $\int \cos^2 x \sin^3 x$

$$\text{let } u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{-du}{\sin x}$$

$$\int \cos^4 x \sin^3 x = \int u^4 \cdot \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$$

$$\int \cos^4 x \sin^3 x = -\int u^4 \cdot \sin x \, du$$

$$\int \cos^4 x \sin^3 x = -\int u^4 \cdot (1 - \cos^2 x) = -\int u^4 \cdot (1 - u^2)$$

$$\int \cos^4 x \sin^3 x = -\int u^4 - u^6 = \int u^6 - u^4$$

$$\int \cos^4 x \sin^3 x = \left[\frac{u^7}{7} - \frac{u^5}{5} \right]$$

$$\int \cos^4 x \sin^3 x = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

3. $\int \cos x \sin^3 x \, dx$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$dx = \frac{du}{\cos x}$$

$$\int \cos x \sin^3 x \, dx = \int \frac{\cos x \cdot u^3 \, du}{\cos x} = \int u^3 \, du$$

$$\int \cos x \sin^3 x \, dx = \left[\frac{u^4}{4} + C \right]$$

$$\int \cos x \sin^3 x \, dx = \left[\frac{\sin^4 x}{4} + C \right]$$