① $\int \sin^6 x\, dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)\left(1-\cos 2x\right)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1 - \cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$

$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + \{2(2\cos^2 2x) - \frac{1}{2} 2\cos 4x \cos 2x$

$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right]$

$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right]$

$= \frac{1}{32}\left[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right]$

$= \frac{1}{32}[10 - 15\cos 2x + 6\cos 4x - \cos 6x]$

Let $\sin^6 x = R$

$R = \frac{1}{32}\int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)\, dx$

$R = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$

$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$