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Matric Number: 19/MHS 01/373

Course code: MAT 104

Assignment

1. $\int \sin^6 x \, dx = \int (\sin^2 x)^3$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$(\sin^2 x)^3 = \left(\frac{1 - \cos 2x}{2}\right)^3 = \frac{1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x}{8}$

$\int \sin^6 x \, dx = \frac{1}{8}\int (1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x)\, dx$

$= \frac{1}{8}\left[\int 1\, dx - 3\int \cos 2x \, dx + 3\int \cos^2 2x \, dx - \int \cos^3 2x \, dx\right]$

$= \frac{1}{8}\left[x - 3\int \cos u \cdot \frac{du}{2} + 3\int \left(\frac{1 + \cos 4x}{2}\right) dx - \int (\cos^2 2x)(\cos 2x) dx\right]$

$= \frac{1}{8}\left[x - \frac{3}{2}\sin u + 3\left(\int \frac{1}{2} dx + \int \frac{\cos 4x}{2} dx\right) - \left(\int (1 - \sin^2 2x)(\cos 2x) dx\right)\right]$

$= \frac{1}{8}\left[x - \frac{3}{2}\sin 2x + 3\left(\frac{x}{2} + \frac{\sin 4x}{8}\right) - \int (\cos 2x - \sin^2 2x \cos 2x) dx\right]$

$= \frac{1}{8}\left[x - \frac{3}{2}\sin 2x + \frac{3}{2}x + \frac{3}{8}\sin 4x - \left(\int \cos 2x \, dx - \int \sin^2 2x \cos 2x \, dx\right)\right]$

$= \frac{1}{8}\left[x - \frac{3}{2}\sin 2x + \frac{3}{2}x + \frac{3}{8}\sin 4x - \left(\frac{\sin 2x}{2} - \int u^2 \cdot \cancel{\cos 2x} \cdot \frac{du}{2\cancel{\cos 2x}}\right)\right]$

$= \frac{1}{8}\left[x - \frac{3}{2}\sin 2x + \frac{3}{2}x + \frac{3}{8}\sin 4x - \frac{\sin 2x}{2} + \frac{1}{2}\int u^2 du\right]$

$= \frac{1}{8}\left[\left(x + \frac{3}{2}x\right) - 2\sin 2x + \frac{3}{8}\sin 4x + \frac{1}{2}\left[\frac{u^3}{3}\right]\right] + C$

$= \frac{1}{8}\left[\frac{5}{2}x - 2\sin 2x + \frac{3}{8}\sin 4x + \frac{u^3}{6}\right] + C$

$= \frac{1}{8}\left[\frac{5}{2}x - 2\sin 2x + \frac{3}{8}\sin 4x + \frac{\sin^3 2x}{6}\right] + C$

$= \frac{5}{16}x - \frac{\sin 2x}{4} + \frac{3}{64}\sin 4x + \frac{\sin^3 2x}{48} + C$

2. $\int \cos^4 x \sin^3 x \, dx$

$u = \cos x \; ; \; du/dx = -\sin x \; ; \; dx = \frac{-du}{\sin x}$

$= \int u^4 \overset{\sin^2 x}{\cancel{\sin^3 x}} \cdot \frac{-du}{\cancel{\sin x}}$

$= -\int u^4 \cdot \sin^2 x \; du$

But $\sin^2 x = 1 - \cos^2 x$

$= -\int u^4 (1 - \cos^2 x) \, du$

But $\cos x = u$

$= -\int u^4 (1 - u^2) \, du$

$= -\int (u^4 - u^6) \, du$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + c$

$= \frac{u^7}{7} - \frac{u^5}{5} + c$

$\int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

or

$\frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c$

3. $\int \cos x \sin^3 x \, dx$

Let $u = \sin x$; $du/dx = \cos x$; $dx = \dfrac{du}{\cos x}$

$= \int \cancel{\cos x} \cdot u^3 \cdot \dfrac{du}{\cancel{\cos x}}$

$= \int u^3 \, du$

$= \dfrac{u^4}{4} + c$

$\therefore \int \cos x \sin^3 x \, dx = \dfrac{\sin^4 x}{4} + c$