from, $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$\therefore \cos 2x \cos 4x = \frac{1}{2}\left[\cos 6x + \cos 2x\right]$

$$\int \cos^3 2x \, dx = \frac{1}{2}\left[\int \cos 2x + \frac{1}{2}(\cos 6x + \cos 2x)\, dx\right]$$

$$= \frac{1}{2}\int \cos 2x\, dx + \frac{1}{4}\int \cos 6x \, dx + \frac{1}{4}\int \cos 2x \, dx$$

$$= \frac{1}{2}\cdot\frac{\sin 2x}{2} + \frac{1}{4}\cdot\frac{\sin 6x}{6} + \frac{1}{4}\cdot\frac{\sin 2x}{2}$$

$$\int \cos^3 2x \, dx = \frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8}$$

$$\int \sin^6 x \, dx = \frac{1}{8}\left[\int dx - 6\int \cos 2x \, dx + 6\int (1 + \cos 4x)\, dx - 8\int \cos^3 2x \, dx\right.$$

$$= \frac{1}{8}\left[x - \frac{\cancel{6}^{3}\sin 2x}{2} + 6x + \frac{\cancel{6}^{3}\sin 4x}{4} - 8\left(\frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8}\right)\right]$$

$$= \frac{1}{8}x - \frac{3}{8}\sin 2x + \frac{3}{4}x + \frac{3\sin 4x}{16} - \frac{\sin 2x}{4} - \frac{\sin 6x}{24} - \frac{\sin 2x}{8} + C$$

$$= \frac{7x}{8} - \frac{6\sin 2x}{8} + \frac{3\sin 4x}{16} - \frac{\sin 6x}{24} + C$$

$$\therefore \int \sin^6 x \, dx = \frac{7x}{8} - \frac{6\sin 2x}{8} + \frac{3}{16}\sin 4x - \frac{\sin 6x}{24} + C$$

2. $\int \cos^4 x \sin^3 x \, dx$

Solution

Since m is odd, $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$dx = -\frac{du}{\sin x}$$

1. $\int \sin^6 x \, dx \quad = \int (\sin$

<u>Solution</u>

$$\int \sin^6 x\, dx = \int (\sin^2 x)^3 dx$$

$$= \int \left[\frac{1-2\cos 2x}{2}\right]^3 dx$$

$$= \frac{1}{8}\int (1-2\cos 2x)^3 = \frac{1}{8}\int (1-2\cos 2x)^2 (1-2\cos 2x)\, dx$$

$$= \frac{1}{8}\int (1-4\cos 2x + 4\cos^2 2x)(1-2\cos 2x)\, dx$$

$$= \frac{1}{8}\int (1-4\cos 2x + 4\cos^2 2x - 2\cos 2x + 8\cos^2 2x - 8\cos^3 2x)$$

$$= \frac{1}{8}\int \left[1 - 6\cos 2x + 12\cos^2 2x - 8\cos^3 2x\right] dx$$

$$= \frac{1}{8}\int \left[1 - 6\cos 2x + 12\left(\frac{1+\cos 4x}{2}\right) - 8\cos^3 2x\right] dx$$

$$= \frac{1}{8}\left[\int dx - 6\int \cos 2x\, dx + 6\int (1+\cos 4x)\, dx - 8\int \cos^3 2x\, dx\right]$$

but $\int \cos^3 2x\, dx = \int \cos^2 2x \cdot \cos 2x \, dx$

$$= \int \cos 2x \, \frac{(1+\cos 4x)}{2}\, dx$$

$$= \frac{1}{2}\int \left[\cos 2x + \cos 2x \cos 4x\right] dx$$

$$\int \cos^4 x \sin^3 x\,dx = \int \cos^4 x \sin x \cdot \sin^2 x\,dx$$

$$= \int \cos^4 x \sin x (1-\cos^2 x)\,dx$$

$$= \int u^4 \sin x (1-u^2)\,\frac{-du}{\sin x}$$

$$= -\int u^4(1-u^2)\,du$$

$$= -\int (u^4 - u^6)\,du$$

$$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$$

$$= -\frac{u^5}{5} + \frac{u^7}{7} + c$$

$$\therefore \int \cos^4 x \sin^3 x\,dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C.$$

3. $\int \cos x \sin^3 x\,dx$.

Solution

m = odd, $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{-du}{\sin x}$$

$$\int \cos x \sin^3 x\,dx = \int \cos x \cdot \sin x(1-\cos^2 x)\,\frac{-du}{\sin x}$$

$$= -\int u(1-u^2)\,du$$

$$= -\int (u - u^3)\,du = \int (u^3 - u)\,du$$

$$= \frac{u^4}{4} - \frac{u^2}{2} + c$$

$$\therefore \int \cos x \sin^3 x\,dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + c.$$

$$\int \cos x \sin^3 x\,dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + c$$