

19/MHS01/397

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MATIC NUMBER: 19/MHS01/397

SERIAL NUMBER: 155

DEPARTMENT: MEDICINE AND SURGERY

$$1. \int \sin^6 x \, dx = \int (\sin^2 x)^3 \, dx$$

Solution

$$\int \sin^6 x \, dx = \int (\sin^2 x)^3 \, dx$$

$$= \int \left(\frac{1 - \cos 2x}{2} \right)^3 \, dx$$

$$= \frac{1}{8} \int (1 - \cos 2x)^3 \, dx = \frac{1}{8} \int (1 - \cos 2x)^2 (1 - \cos 2x) \, dx$$

$$= \frac{1}{8} \int (1 - 4\cos 2x + 4\cos^2 2x) (1 - \cos 2x) \, dx$$

$$= \frac{1}{8} \int (1 - 4\cos 2x + 4\cos^2 2x - 2\cos 2x + 8\cos^2 2x - 8\cos^3 2x) \, dx$$

$$= \frac{1}{8} \int [1 - 6\cos 2x + 12\cos^2 2x - 8\cos^3 2x] \, dx$$

$$= \frac{1}{8} \int \left[1 - 6\cos 2x + 12 \left(\frac{1 + \cos 4x}{2} \right) - 8\cos^3 2x \right] \, dx$$

$$= \frac{1}{8} \left[\int dx - 6 \int \cos 2x \, dx + 6 \int (1 + \cos 4x) \, dx - 8 \int \cos^3 2x \, dx \right]$$

$$\text{but } \int \cos^3 2x \, dx = \int \cos^2 2x \cdot \cos 2x \, dx$$

$$= \int \frac{\cos 2x (1 + \cos 4x)}{2} \, dx$$

$$= \frac{1}{2} \int [\cos 2x + 2\cos 2x \cos 4x] \, dx$$

from, $\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$

$$\therefore \cos 2x \cos 4x = \frac{1}{2} [\cos 6x + \cos 2x]$$

$$\begin{aligned} \int \cos^3 2x \, dx &= \frac{1}{2} \left[\int \cos 2x + \frac{1}{2} (\cos 6x + \cos 2x) \, dx \right] \\ &= \frac{1}{2} \int \cos 2x \, dx + \frac{1}{4} \int \cos 6x \, dx + \frac{1}{4} \int \cos 2x \, dx \\ &= \frac{1}{2} \cdot \frac{\sin 2x}{2} + \frac{1}{4} \cdot \frac{\sin 6x}{6} + \frac{1}{4} \cdot \frac{\sin 2x}{2} \end{aligned}$$

$$\therefore \int \cos^3 2x \, dx = \frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8}$$

$$\begin{aligned} \therefore \int \sin^6 x \, dx &= \frac{1}{8} \left[\int dx - 6 \int \cos 2x \, dx + 6 \int (1 + \cos 4x) \, dx - 8 \int \cos^3 2x \, dx \right] \\ &= \frac{1}{8} \left[x - \frac{3 \sin 2x}{2} + 6x + \frac{3 \sin 4x}{4} - 8 \left(\frac{\sin 2x}{4} + \frac{\sin 6x}{24} + \frac{\sin 2x}{8} \right) \right] \\ &= \frac{1}{8} x - \frac{3 \sin 2x}{8} + \frac{3}{4} x + \frac{3 \sin 4x}{16} - \frac{\sin 2x}{4} - \frac{\sin 6x}{24} - \frac{\sin 2x}{8} + C \\ &= \frac{7x}{8} - \frac{6 \sin 2x}{8} + \frac{3 \sin 4x}{16} - \frac{\sin 6x}{24} + C \end{aligned}$$

$$\therefore \int \sin^6 x \, dx = \frac{7x}{8} - \frac{6 \sin 2x}{8} + \frac{3 \sin 4x}{16} - \frac{\sin 6x}{24} + C$$

2. $\int \cos^4 x \sin^3 x \, dx$

Solution

Since m is odd, $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$dx = -\frac{du}{\sin x}$$

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$$\begin{aligned}
 \int \cos^4 x \sin^3 x \, dx &= \int \cos^4 x \sin x \cdot \sin^2 x \, dx \\
 &= \int \cos^4 x \sin x (1 - \cos^2 x) \, dx \\
 &= \int u^4 \sin x (1 - u^2) \frac{-du}{\sin x} \\
 &= - \int u^4 (1 - u^2) \, du \\
 &= - \int (u^4 - u^6) \, du \\
 &= - \left[\frac{u^5}{5} - \frac{u^7}{7} \right] + C \\
 &= - \frac{u^5}{5} + \frac{u^7}{7} + C
 \end{aligned}$$

$$\therefore \int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C.$$

3. $\int \cos x \sin^3 x \, dx$ -

Solution
both ^{are} ~~not~~ odd, $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{-du}{\sin x}$$

$$\begin{aligned}
 \int \cos x \sin^3 x \, dx &= \int \cos x \cdot \sin x (1 - \cos^2 x) \frac{-du}{\sin x} \\
 &= - \int u (1 - u^2) \, du \\
 &= - \int (u - u^3) \, du = \int (u^3 - u) \, du \\
 &= \frac{u^4}{4} - \frac{u^2}{2} + C
 \end{aligned}$$

$$\therefore \int \cos x \sin^3 x \, dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C.$$

$$\int \cos x \sin^3 x \, dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C$$