2. $\cos^4 x \sin^3 x$

$u = \cos x$

$dx = -du/\sin x$

$\therefore \int \cos^4 x \sin^3 x \, dx = -\int u^4 \cdot \sin x \cdot \sin^2 x \frac{du}{\sin x}$

$I = -\int u^4 \cdot \sin^2 x \, du = -\int u^4 \cdot (1-\cos^2 x) \, du$

$I = -\int u^4 \cdot (1-u^2) \, du$

$I = -\int u^4 - u^6 \, du$

$I = -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + c = \frac{-u^5}{5} + \frac{u^7}{7} + c$

$I = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c$

8. $\cos x \sin^3 x$

$u = \cos x \,, \; dx = -du/\sin x$

$$\therefore \int \cos x \sin^3 x \, dx = I = \int u \cdot \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$$

$$I = -\int u \cdot (1 - \cos^2 x) \, du = -\int u \cdot (1 - u^2) \, du$$

$$I = -\int u - u^3 \, du$$

$$I = -\left[\frac{u^2}{2} - \frac{u^4}{4}\right] + c = -\frac{u^2}{2} + \frac{u^4}{4} + c$$

$$I = \frac{(\cos x)^4}{4} - \frac{(\cos x)^3}{2} + c$$

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1. $\sin^6 x = [\sin^2 x]^2 [\sin^2 x]$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\sin^6 x = \left[\frac{1 - \cos 2x}{2}\right]^2 [\sin^2 x]$

$\sin^6 x = \frac{1}{4}[1 - \cos 2x]^2 [\sin^2 x]$

$\sin^6 x = \frac{1}{4}[1 - 2\cos 2x + \cos^2 2x]\left[\frac{1 - \cos 2x}{2}\right]$

$\cos^2 2x = \frac{1 + \cos 4x}{2}$

$\therefore \sin^6 x = \frac{1}{4}\left[1 - 2\cos 2x + \frac{1 + \cos 4x}{2}\right]\left[\frac{1 - \cos 2x}{2}\right]$

$= \frac{1}{4}\left[1 - 2\cos 2x + \frac{1}{2} + \frac{\cos 4x}{2}\right]\left[\frac{1}{2}\left\{\frac{1}{2} - \frac{\cos 2x}{2}\right\}\right]$

$= \frac{1}{4}\left[\frac{1}{2} - \frac{\cos 2x}{2} - \cos 2x + \cos^2 2x + \frac{1}{4} - \frac{\cos 2x}{4} + \frac{\cos 4x}{4} - \frac{\cos 4x . \cos 2x}{4}\right]$

$= \frac{1}{4}\left[\frac{3}{4} - \frac{7\cos 2x}{4} + \frac{1 + \cos 4x}{2} + \frac{\cos 4x}{4} - \frac{\cos 4x . \cos 2x}{4}\right]$

$= \frac{1}{4}\left[\frac{5}{4} - \frac{7\cos 2x}{4} + \frac{3\cos 4x}{4} - \left[\frac{\frac{1}{2}[\cos 6x + \cos 2x]}{4}\right]\right]$

$= \frac{1}{4}\left\{\frac{5}{4} - \frac{15\cos 2x}{8} + \frac{3\cos 4x}{4} - \frac{\cos 6x}{8}\right]$

1. cont'd

$$\sin^6 x = \frac{1}{4}\left[\frac{1}{8}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]\right]$$

$$\int \sin^6 x \, dx = \frac{1}{32}\int 10 - 15\cos 2x + 6\cos 4x - \cos 6x$$

$$\int \sin^6 x \, dx = \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + c$$

$$\int \sin^6 x \, dx = \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + c$$

or

$$\int \sin^6 x \, dx = \frac{1}{32}\left[\frac{1}{6}\left[60x - 45\sin 2x + 9\sin 4x - \sin 6x\right] + c\right.$$

$$\int \sin^6 x \, dx = \frac{1}{192}\left[60x - 45\sin 2x + 9\sin 4x - \sin 6x\right] + C$$