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(i) $\int \sin^6 x \, dx$

solution

$$\int \sin^6 x = \int \sin^2 x - \sin^4 x \, dx$$

$$\int \sin^6 x = \int 1 - \cos 2x \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$\sin^6 \int \sin^6 x = \int \frac{1-\cos 2x}{2} \cdot 1 - 2\cos 2x + \cos^2 2x \, dx$$

$$\int \sin^6 x = \frac{1}{8}\int 1 - 3\cos 2x + \cos^2 2x - \cos 2x + 2\cos^2 2x - \cos 2x \, dx$$

$$\int \sin^6 x = \frac{1}{8}\int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$$

$$\int \sin^6 x = \frac{1}{8}\int 1 - 3\cos 2x + 3\left(\frac{1+\cos 4x}{2}\right) \cancel{-\cos 2x}$$

$$- \cos 2x (1 - \sin^2 x) \, dx$$

$$\int \sin^6 x = \frac{1}{8}\int 1 - 3\cos 2x + \frac{3}{2} + 3\frac{\cos 4x}{2} - \cos 2x$$

$$+ \cos 2x \sin^2 \, dx$$

$$\int \sin 6x = \frac{1}{8}\int \frac{5}{2} - 4\cos 2x + 3\frac{\cos 4x}{2} +$$

$$\cos 2x \sin^2 2x \, dx$$

$$\int \sin 6x = \frac{1}{8}\left(\frac{5}{2}x - \frac{4\sin 2x}{2} + 3\frac{\sin 4x}{2} \times 4\right.$$

$$+\frac{\sin^3 2x}{6}\Big] + C$$

2.) $\int \cos^4 x \sin^3 x$

Let $u = \cos x$

$\frac{du}{dx} = \sin x$

$dx = \frac{du}{\sin x}$

$\int \cos^4 x \sin^3 x = \int u^4 \cdot \sin x \cdot \sin^2 x$

$\int \cos^4 x \sin^3 x = -\int u^4 \cdot \sin x \, du$

$\int \cos^4 x \sin^3 x = -\int u^4 \cdot (1 - \cos^2 x)$

$\int \cos^4 x \sin^3 x = -\int u^4 - u^6 = \int u^6 - u^4$

$\int \cos^4 x \sin^3 x = \left[\frac{u^7}{7} - \frac{u^5}{5}\right]$

$\int \cos^4 x \sin^3 x = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3.) $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = du/\cos x$

$\int \cos x \sin^3 x \, dx = \int \cos x \frac{u^3 du}{\cos x}$

$\sin 6x = \frac{1}{8}\left(\frac{5}{2}x - \frac{4\sin 2x}{2} + \frac{3\sin 4x}{2} \times \frac{1}{4}\right) +$

$\int \cos 2x \sin^2 2x \, dx$

$\int \cos 2x \sin^2 2x \, dx = \int \frac{\cancel{\cos 2x}(u)^2 \, du}{2 \cancel{\cos 2x}}$ 2.)

let $u = \sin 2x$

$\frac{du}{dx} = 2\cos 2x$

$dx = \frac{du}{2\cos 2x}$

$= \int u^2 \frac{du}{2}$

$= \frac{1}{2}\int u^2 du$

$= \frac{1}{2}\left(\frac{u^3}{3} + c\right)$

$\frac{1}{2}\left(\frac{\sin^3 2x}{3} + c\right.$

$\therefore \cancel{\int \sin 6x = \frac{1}{8}}\Big($

$\therefore \int \sin 6x = \frac{1}{8}\left[\frac{5x}{2} \cancel{+} \frac{2\sin 2x}{2} + \frac{3\sin 4x}{8}\right.$ 3.)

$\therefore \int \sin 6x = \frac{1}{8}\left[\frac{5}{2}x - 2\sin 2x + \frac{3\sin 4x}{8}\right.$

$$\int \cos x \sin^3 x \, dx = \left[\frac{u^4}{4} + c\right]$$

$$\int \cos x \sin^3 x \, dx = \left[\frac{\sin^4 x}{4} + c\right]$$