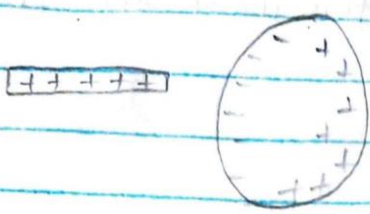
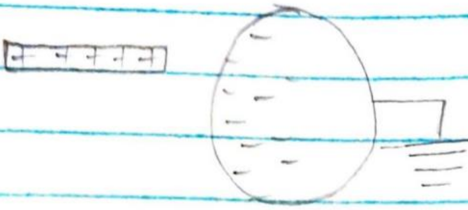
(6.) Producing a negatively charged on a sphere by induction

(i.)



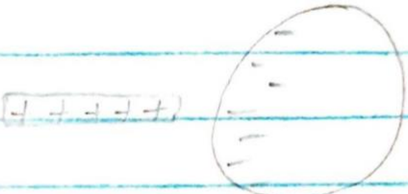
A positively charged rod is brought near a neutral conducting sphere. The protons are forced to move from the left side of the sphere to the right side.

(ii)



A "ground" is brought ~~here~~ near and touches the right side of the sphere. The unbalance of charge is neutralized as the protons leave the sphere and pass through the "ground".

(iii)



The right side of the sphere has now been neutralized by the departure of ~~electrons~~ protons. There remains an unbalance of charge on the left side of the sphere.

As the rod is pulled away, there is a movement of the remaining electrons within the conducting sphere which results in a uniform distribution of the negative charges throughout the sphere's surface.

(b) $q_1 + q_2 = 5.0\times10^{-5}\,C$

$F = \dfrac{kq_1q_2}{r^2}$

$10 = \dfrac{9\times10^{9}\times q_1q_2}{4}$

$q_1q_2 = 4.44\times10^{-10}\,C^2 \;.\,.\,.\,.\,.\; (i)$

$q_1 + q_2 = 5.0\times10^{-5}\,C \;.\,.\,.\,.\,.\; (ii)$

from eqn (ii)

$q_1 = 5.0\times10^{-5} - q_2 \;.\,.\,.\,.\; (iii)$

Sub eqn (iii) into eqn (i)

$(5.0\times10^{-5} - q_2)\times q_2 = 4.44\times10^{-10}$

$5.0\times10^{-5}q_2 - q_2^{\,2} = 4.44\times10^{-10}$

$5.0\times10^{-5}q_2 - q_2^{\,2} - 4.44\times10^{-10}$

$q_2^{\,2} - 5.0\times10^{-5}q_2 + 4.44\times10^{-10} = 0$

Using the quadratic formula

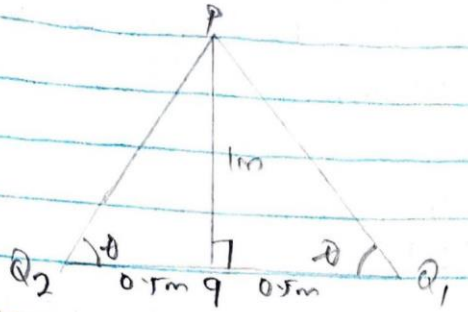
$q = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-5.0\times10^{-5}) \pm \sqrt{(5.0\times10^{-5})^2 - 4(1)(4.44\times10^{-10})}}{2(1)}$

$= \dfrac{5.0\times10^{-5} + \sqrt{7.24\times10^{-10}}}{2}$ or $\dfrac{5.0\times10^{-5} - \sqrt{7.24\times10^{-10}}}{2}$

$q = 3.85 \times 10^{-5}\,C$ $\qquad$ $q = 1.15 \times 10^{-5}\,C$

(c.)



$Q_1 = Q_2 = 8 \times 10^{-6} C$

$\tan\theta = \frac{opp}{adjacent}$

$\theta = \tan^{-1}\left(\frac{1}{0.5}\right)$

$\theta = 63.43°$

Using pythagoras theorem

$|PQ|^2 = |PO|^2 + |OQ|^2$

$|PQ_1| = \sqrt{(1)^2 + (0.5)^2}$

$\therefore |PQ_1| = 1.12$

$|PQ_1| = |PQ_2| = 1.12$

$E_1 = \frac{KQ_1}{r^2} = \frac{9 \times 10^9 Nm^2/C^2 \times 8 \times 10^{-6} C}{(1.12m)^2} = 5.74 \times 10^4 N/C.$

$E_2 = \frac{KQ_2}{r^2} = \frac{9 \times 10^9 Nm^2/C^2 \times 8 \times 10^{-6} C}{(1.12m)^2} = 5.74 \times 10^4 N/C.$

$E_3 = \frac{KQ_3}{r^2} = \frac{9 \times 10^9 Nm^2/C^2 \times q}{(1)^2} = 9 \times 10^9 q$

E	θ	Fx	Fy
5.74×10^4	$63.43°$	$5.74 \times 10^{-4} \cos 63.43°$	$5.74 \times 10^{-4} \sin 63.43°$
5.74×10^4	$63.43°$	$5.74 \times 10^{-4} \cos 63.43°$	$5.74 \times 10^{-4} \sin 63.43°$
$9 \times 10^9 q$	$90°$	0	$9 \times 10^9 q \sin 90°$
		$\Sigma fx = 0$	$1.03 \times 10^{-3} + 9 \times 10^9 q$

$|R| = \sqrt{\Sigma fx^2 + \Sigma fy^2}$

$0 = \sqrt{(0)^2 + (1.03 \times 10^{3} + 9 \times 10^9 q)^2}$

$0 = \sqrt{(1.03 \times 10^{-3} + 9 \times 10^9 q)^{\cancel{2}}}$

$0 = 1.03 \times 10^{-3} + 9 \times 10^9 q$

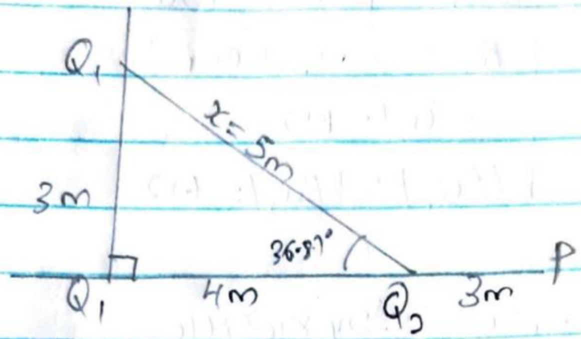
$\frac{-1.03 \times 10^{-3}}{9 \times 10^9} = \frac{\cancel{9 \times 10^9} q}{\cancel{9 \times 10^9}}$ $\quad \therefore q = -11 \times 10^{-6} C = -11 \mu C$

2(a) Electric field is a region of space in which an electric charge experience an electric force.
Electric field intensity (strength) E, can be defined as the force per unit charge. Mathematically, its magnitude can be represented by

$$E = \frac{F\,(N)}{q_0\,(C)}$$

Measured in Newton per coulomb (N/C).

(b)



$$E_1 = \frac{kQ_1}{r^2} = \frac{9\times10^9\,Nm^2/C^2 \times 8\times10^{-9}C}{(7)^2} = 1.47\,N/C$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9\times10^9\,Nm^2/C^2 \times 12\times10^{-9}C}{(3)^2} = 12\,N/C$$

E	θ	Fx	Fy
1.47 N/C	$0°$	$1.47\cos 0°$	$1.47\sin 0°$
12 N/C	$0°$	$12\cos 0°$	$12\sin 0°$
		Σfx 13.47	Σfy 0

$$|R| = \sqrt{\Sigma fx^2 + \Sigma fy^2}$$
$$|R| = \sqrt{(13.47)^2 + (0)^2}$$
$$|R| = \sqrt{(13.47)^2}$$
$$\therefore |R| = 13.47\,N/C$$
$$\therefore E_{net} = 13.5\,N/C$$

Using pythagoras theorem

$x^2 = 3^2 + 4^2$

$x^2 = 9 + 16$

$\sqrt{x^2} = \sqrt{21} = 5m$

$\tan\theta = 3/4$

$\theta = \tan^{-1}(3/4) = 36.87°$

$E_1 = \frac{kQ_1}{r^2} = \frac{(9\times10^9 Nm^2/C^2)\times(8\times10^{-9}C)}{(3)^2} = 8 N/C$

$E_2 = \frac{kQ_2}{r^2} = \frac{(9\times10^9 Nm^2/C^2)\times(12\times10^{-9}C)}{(5)^2} = 4.32 N/C$

E	θ	F_x	F_y
8 N/C	90°	0	$8\sin 90°$
4.32 N/C	36.87°	$4.32\cos 36.87$	$4.32\sin 36.87$
		$\Sigma f_x = 3.46$	$\Sigma f_y = 10.59$

$|R| = \sqrt{\Sigma f_x^2 + \Sigma f_y^2}$

$|R| = \sqrt{(3.46)^2 + (10.59)^2}$

$|R| = 11.14 N/C$

$\therefore E_{net} = 11.14 N/C$

PART B

10 a) Magnetic flux (often denoted ϕ or ϕ_B) through a surface is the surface integral of the normal component of the magnetic field flux density B passing through the surface. The SI unit of magnetic field is Weber (wb; in derived units, volt-seconds and CGS unit is the maxwell).

b) $m = 9.11\times10^{-31} kg$, $r = 1.4\times10^{-7} m$, $B = 3.5\times10^{-1} W/m^2$.

$\therefore \omega = \frac{qB}{m}$

$= \frac{1.60\times10^{-19}C \times 3.5\times10^{-1} W/m^2}{9.11\times10^{-31} kg} = 6.18\times10^{10} rad/s$

4(a) The Biot-Savart Law is based on the following Observations for the magnetic field $d\vec{B}$ at a point P associated with the length element $d\hat{l}$ of a wire carrying a steady current

Magnetic Field of a Straight Current Carrying Conductor

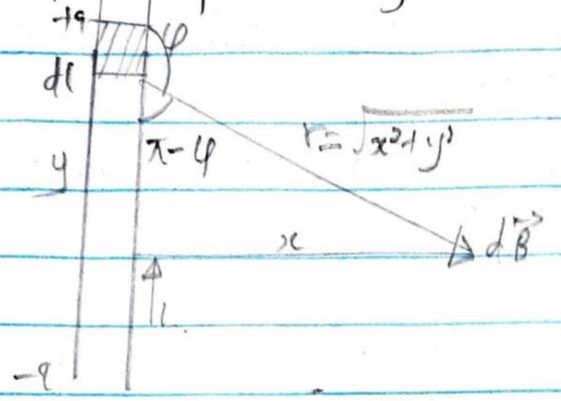


Fig 1 A section of a Straight Current Carrying Conductor

Applying the Biot-Savart Law, we find the magnitude of the field $d\vec{B}$

$$B=\frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin\varphi}{r^2}$$

$$\sin(\pi-\varphi)=\sin\theta$$

$$\therefore B=\frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\varphi)}{r^2}$$

From the diagram $r^2=x^2+y^2$ (Pythagoras theorem)

$$B=\frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\varphi)}{x^2+y^2}\ \ldots\ldots \quad (i)$$

But $\sin(\pi-\varphi)=\dfrac{x}{\sqrt{x^2+y^2}}=\dfrac{x}{(x^2+y^2)^{1/2}}$. (ii)

Substituting eqn (ii) into (i);

$$B=\frac{\mu_0 I}{\cdot}\int^{a}dl\,\frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad \ldots \text{(iii)}$$

Using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Eqn (iii) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2(x^2+y^2)^{1/2}} \right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2(x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P we consider it infinitely long. That is, when a is much larger than x,

$$(x^2+a^2)^{1/2} \simeq a, \text{ as } a \to \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

This defines the magnitude of the magnetic field of flux density B near a long, straight current carrying conductor