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MEDICINE & SURGERY
19/MHS01/093
MAT 104.

1) $\int \sin^6 x \, dx$

$$\int (\sin^2 x)^3 \, dx$$

$$\Rightarrow \int (1-\cos^2 x)^3 \, dx$$

$$\Rightarrow \int -\cos^6 x + 3\cdot\cos^4 x - 3\cos^2 x + 1 \, dx$$

using $\cos 2x = 2\cos^2 x - 1$

$$\Rightarrow \int -\left(\frac{1}{8}\right)\cdot(1+\cos 2x)^3 + 3\left(\frac{1}{4}\right)(1+\cos 2x)^2 - 3\left(\frac{1}{2}\right)(1+\cos 2x) + 1 \, dx$$

$$= \int 1 - \left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)\cdot\cos 2x + \left(\frac{3}{4}\right)(1+\cos^2 2x + 2\cdot\cos 2x) - \left(\frac{1}{8}\right)(\cos^3(2x) + 3\cos^2(2x) + 3\cos(2x) + 1) \, dx$$

$$= \int -\frac{1}{2} - \left(\frac{3}{2}\right)\cos 2x + \frac{3}{4} + \frac{3}{4}\cos^2 2x + \frac{3}{2}\cos 2x - \left(\frac{1}{8}\right)\cos^3 2x - \left(\frac{3}{8}\right)\cos^2 2x - \left(\frac{3}{8}\right)\cos 2x - \frac{1}{8} \, dx$$

$$= \int \frac{1}{8} - \left(\frac{3}{8}\right)\cos 2x + \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos^3 2x \, dx$$

$$= \int \frac{1}{8} - \frac{3}{8}\cos 2x + \left(\frac{3}{16}\right) + \left(\frac{3}{16}\right)\cos 4x - \frac{1}{8}\cos^2 2x \cdot \cos 2x \, dx$$

$$\int \frac{5}{16} - \left(\frac{3}{8}\right)\cos 2x + \left(\frac{3}{16}\right)\cos 4x - \left(\frac{1}{8}\right)\cdot(1-\sin^2 2x)\cdot \cos 2x\,dx$$

$$\int \frac{5}{16} - \frac{3}{8}\cos 2x + \left(\frac{3}{16}\right)\cos 4x\,dx - \frac{1}{8}\int (1-\sin^2 2x)\cos 2x\,dx$$

$$\int \frac{5}{16} - \left(\frac{3}{8}\right)\cos 2x + \left(\frac{3}{16}\right)\cos 4x\,dx.$$

$$\left(\frac{5}{16}\right)x - \left(\frac{3}{16}\right)\sin 2x + \frac{3}{64}\sin 4x.$$

$y = \sin 2x$, $\left(\frac{1}{2}\right)dy = \cos 2x\,dx$

$\frac{1}{2}\cdot y - \left(\frac{1}{6}\right)y^3$

substitute $y = 2x$

$\left(\frac{1}{2}\right)\sin 2x - \left(\frac{1}{6}\right)\sin^3 2x.$

$$\left(\frac{5}{16}\right)\cdot x - \left(\frac{3}{16}\right)\sin 2x + \frac{3}{64}\sin 4x - \frac{1}{16}\sin 2x + \frac{1}{48}\sin^3 2x$$

$$\int \sin^6 x = \frac{60x - 48\sin 2x + 4\sin^3 2x + 9\sin 4x}{192}.$$

2. $\cos^4 x \sin^3 x$

$n = 4 \quad m = 3$

$u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{-du}{\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\int u^4 \cdot \sin^2 x \cdot -\frac{du}{\sin x}$

$\int u^4 . 1 - \cos^2 x \cdot -du.$

$\int u^4 . 1 - u^2 . -du.$

$\int u^4 - u^6 - du$

$\frac{\sin x^5}{5} - \frac{\sin^7 x}{7} + c$

(3) $\cos x \sin 3x$

Recall $\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$= \frac{1}{2}\left[\sin(4x) - \sin(-2x)\right]$

$= \frac{1}{2}\left[-\frac{\cos 4x}{4} + \frac{\cos -2x}{2}\right]$

$$= \frac{1}{2}\left[\frac{-\cos 4x}{4} - \frac{\cos 2x}{2}\right].$$

$$= \frac{-\cos 4x}{8} - \frac{\cos 2x}{4} + C$$