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COLLEGE: MHS

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ASSIGNMENT II

Question

Integrate the following functions

- 1) $\sin^3 x$
- 2) $\cos^3 x \sin^2 x$
- 3) $\cos x \sin^3 x$

Sol.

$$1) \sin^3 x = (\sin^2 x)^2 (\sin x)$$

$$= \left(\frac{1 - \cos 2x}{2} \right)^2 \left(\frac{1 - \cos 2x}{2} \right)$$

$$= \frac{1}{8} (1 - 2\cos 2x + \cos^2 2x) (1 - \cos 2x)$$

$$= \frac{1}{8} (1 - 2\cos 2x + \frac{1 + \cos 4x}{2}) (1 - \cos 2x)$$

$$= \frac{1}{16} ((2 - 4\cos 2x + 1 + \cos 4x) (1 - \cos 2x))$$

$$= \frac{1}{16} \int [3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos$$

$$= \frac{1}{16} \int [3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos$$

$$= \frac{1}{32} \int [10 - 15\cos 2x + 6\cos 4x - \cos 6x]$$

$$= \frac{1}{32} \int [10 - 15\cos 2x + 6\cos 4x - \cos 6x] dx.$$

$$= \frac{1}{32} \left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6} \right] + C$$

$$= \frac{1}{32} \left[10x - \frac{15 \sin^2 x}{2} + \frac{3 \sin^4 x}{2} - \frac{\sin^6 x}{6} \right] + C$$

$$= \frac{1}{192} [60x - 45 \sin^2 x + 9 \sin^4 x - \sin^6 x] + C$$

2) $\cos^4 x \sin^2 x$

let $u = \cos x$

$$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{-du}{\sin x}$$

$$\int u^4 \cdot \sin x \cdot \sin^2 x \cdot \frac{-du}{\sin x}$$

$$= -\int u^4 \cdot \sin^2 x \cdot du$$

$$= -\int u^4 \cdot (1 - \cos^2 x) du$$

$$= -\int u^4 \cdot (u^2 - 1) du$$

$$= \int (u^6 - u^4) du$$

$$= \left[\frac{u^7}{7} - \frac{u^5}{5} \right] + C$$

$$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

3) $\cos x \sin^3 x$

$u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x dx$$

$$\int u^3 \cdot du$$

$$= \left[\frac{u^4}{4} \right] + C$$

$$\Rightarrow \frac{\sin^4 x}{4} + C$$