CHORIO RUAMAH EMMANUEL

19/MHS01/128

1 $\int \sin^6 x \, dx$

$= \int (\sin^2 x)^3 \, dx$

$= (1 - \cos^2 x)^3 \, dx$

$= \int (1 - \cos^2 x)(1 - 2\cos^2 x + \cos^4 x) \, dx$

$= \int (1 - 2\cos^2 x + \cos^4 x - \cos^2 x + \cdots$

$= \int (1 - 3\cos^2 x + \cos^4 x - \cos^6 x) \, dx$

$= \int 1 - 3\int \cos^2 x + \int \cos^4 x - \int \cos^6 x$

$= x - 3\left(\frac{\cos^3 x}{3}\right) + \frac{\cos^5 x}{5} - \frac{\cos^7 x}{7}$

$= x - \cos^3 x + \frac{\cos^5 x}{5} - \frac{\cos^7 x}{5}$

2) $\int \cos^4 x \sin^3 x$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$= \int u^4 \sin^3 x \cdot \frac{-du}{\sin x}$

$= \int u^4 \sin^2 x \cdot -du$

$= \int u^4 (1 - \cos^2 x) \, du$

$= \int u^4 (1 - u^2) \, du$

$= \int (u^4 - u^6) \, du$

$= \frac{u^{4+1}}{4+1} - \frac{u^{6+1}}{6+1}$

$= \frac{u^5}{5} - \frac{u^7}{7} + C$

$= \frac{(\cos x)^5}{5} - \frac{(\cos x)^7}{7} + C$

3) $\int \cos^x \sin^3 x$

$u = \cos^x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

$dx = \frac{-du}{\sin x}$

$\Rightarrow \int u \cdot \sin^3 x \cdot \frac{-du}{\sin x}$

$= -\int u \cdot \sin^2 x \cdot du$

$= -\int \frac{u^2}{2} \cdot$

$\int u \cdot \sin^3 x \cdot \frac{-du}{\sin x}$

$= -\int u \sin^2 x \, du$

$= -\int u(1 - \cos^2 x) du$

$= -\int u(1 - u^2) du$

$= -\int (u - u^3) du = -\int (u - u^3) du$

$= -\left(\frac{u^2}{2} - \frac{u^4}{4}\right) + C$

$= \frac{-u^2}{2} + \frac{u^4}{4} + C$

$\Rightarrow \frac{u^4}{4} - \frac{u^2}{2} + C$

$\Rightarrow \frac{(\cos x)^4}{4} - \frac{(\cos x)^2}{2} + C$