

SHERIFF FATIMA

19/MHS 01/398

MBBS

①  $\int \sin^6 x \, dx$

$$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$$

$$= \left( \frac{1 - \cos 2x}{2} \right)^2 \left( \frac{1 - \cos 2x}{2} \right)$$

$$= \frac{1}{8} (1 - 2\cos 2x + \cos^2 2x) (1 - \cos 2x)$$

$$= \frac{1}{8} \left( \frac{1 - 2\cos 2x + 1 + \cos 4x}{2} \right) (1 - \cos 2x)$$

$$= \frac{1}{16} (2 - 4\cos 2x + 1 + \cos 4x) (1 - \cos 2x)$$

$$= \frac{1}{16} (3 - 4\cos 2x + \cos 4x) (1 - \cos 2x)$$

$$= \frac{1}{16} (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$$

$$= \frac{1}{16} [3 - 7\cos 2x + \cos 4x + \{2(2\cos^2 2x) - \frac{1}{2}2\cos 4x \cos 2x\}]$$

$$= \frac{1}{16} \left[ 3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x) \right]$$

$$= \frac{1}{16} \left[ 3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x) \right]$$

$$= \frac{1}{32} [6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x]$$

$$= \frac{1}{32} [10 - 15\cos 2x + 6\cos 4x - \cos 6x]$$

Let  $\sin^2 x = R$

$$R = \int \frac{1}{32} (10 - 15\cos 2x + 6\cos 4x - \cos 6x) \, dx$$

$$R = \frac{1}{32} \left( \frac{10x}{2} - \frac{15\sin 2x}{4} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6} \right) + C$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$$

②  $\int \cos^4 x \sin^3 x \, dx$

$$u = \cos x, \quad \frac{du}{dx} = -\sin x, \quad dx = \frac{-du}{\sin x}$$

$$\int \cos^4 x \sin^3 x \, dx = \int u^4 \cdot \sin x \cdot \sin^2 x \frac{du}{-\sin x} = - \int u^4 \sin^2 x \, du$$

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191MHS011398

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$$\int (\cos^4 x \sin^3 x) dx = - \int u^4 (1 - u^2) du = - \int u^4 (1 - u^2) du$$

$$= - \int (u^4 - u^6) du$$
$$\int (\cos^4 x \sin^3 x) dx = - \left( \frac{u^5}{5} - \frac{u^7}{7} \right) + C = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$\int (\cos^4 x \sin^3 x) dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

③  $\int (\cos x \sin^3 x) dx$

let  $u = \sin x \rightarrow \frac{du}{dx} = \cos x \rightarrow du = \cos x dx$

$$\int \cos x \sin^3 x dx = \int u^3 du = \frac{u^4}{4} + C = \frac{\sin^4 x}{4} + C$$