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MEDICINE AND SURGERY
19/MHS01/130
MAT 104

1 $\int \sin^6 x\,dx$

Recall $\sin^2 x = 1 - \cos^2 x$

$\therefore \int \sin^6 x\,dx$

$= \int (\sin^2 x)^3\,dx$

$= \int (1 - \cos^2 x)^3\,dx$

$= \int 1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x\,dx$

$= x - 3 \times \frac{1}{2}\left[\frac{\sin 2x}{2} + x\right] + 3 \times \frac{1}{4}\left[\frac{3x}{2} + \sin 2x + \frac{1}{8}\sin 4x\right] - \left[\frac{5x}{16} + \frac{1}{4}\sin 2x + \frac{3}{64}\sin 4x - \frac{1}{48}\sin^3(2x)\right] + C$

$= x - \frac{3}{2}\left[\frac{\sin 2x}{2} + x\right] + \frac{3}{4}\left[\frac{3x}{2} + \sin 2x + \frac{\sin 4x}{8}\right] - \left[\frac{5x}{16} + \frac{\sin 2x}{4} + \frac{3\sin 4x}{64} - \frac{\sin^3(2x)}{48}\right] + C$

$= x - \frac{3\sin 2x}{4} - \frac{3x}{2} + \frac{9x}{8} + \frac{3\sin 2x}{4} + \frac{3\sin 4x}{32} - \frac{5x}{16} - \frac{\sin 2x}{4} - \frac{3\sin 4x}{64} + \frac{\sin^3(2x)}{48} + C$

$= x - \frac{3\sin 2x}{4} + \frac{3\sin 2x}{4} - \frac{\sin 2x}{4} - \frac{3x}{2} + \frac{9x}{8} - \frac{5x}{16} + \frac{3\sin 4x}{32} - \frac{3\sin 4x}{64} + \frac{\sin^3(2x)}{48} + C$

$$\frac{dp}{du} = 2u - 4u$$

$$du = \frac{dp}{2u - 4u}$$

$$\therefore \cancel{\int u^2 = 0} \quad \therefore \int (u^2 - u^4)^{3/2}\, du$$

$$= \int p^{3/2} \times \frac{dp}{2u - 4u}$$

$$= \frac{p^{\frac{3}{2}+1}}{\frac{3}{2}+1} \times \frac{1}{2u - 4u} + c$$

$$= \frac{p^{5/2}}{5/2} \times \frac{1}{2u - 4u} + c$$

$$= \frac{2p^{5/2}}{5} \times \frac{1}{2u - 4u} + c$$

$$= \frac{2[u^2 - u^4]^{5/2}}{5[2u - 4u]} + c$$

$$= \frac{2[u^2 - u^4]^{5/2}}{10u - 20u} + c$$

$$= \frac{2[\sin^2 x - \sin^4 x]^{5/2}}{10\sin x - 20\sin x} + c$$

$$= \frac{2[\sin^2 x - \sin^4 x]^{5/2}}{2[5\sin x - 10\sin x]} + c$$

$$= \frac{[\sin^2 x - \sin^4 x]^{5/2}}{-5\sin x} + c$$

$$\therefore \int \cos^4 x \sin^3 x\, dx$$

$$= \frac{[\sin^2 x - \sin^4 x]^{5/2}}{-5\sin x} + c$$

3 $\int \cos x \sin^3 x \, dx$

when both powers are odd, let

$u = \cos x$

$\frac{du}{dx} = -\sin x$, $\quad -du = \sin x \, dx$

$du = -\sin x \, dx$

$dx = \frac{du}{-\sin x} = \frac{-du}{\sin x}$

$\therefore \int \cos x \sin^3 x \, dx$

$= \int u \sin^3 x \cdot \frac{-du}{\sin x}$

$= -\int u \sin^2 x \, du$

but $\sin^2 x = 1 - \cos^2 x$

$\therefore -\int u \sin^2 x \, du$

$= -\int u (1 - \cos^2 x) \, du$

$= -\int u (1 - u^2) \, du$

$= -\int u - u^3 \, du$

$= \int -u + u^3 \, du$

$= \frac{-u^{1+1}}{1+1} + \frac{u^{3+1}}{3+1} + C$

$= \frac{-u^2}{2} + \frac{u^4}{4} + C$

$\therefore \int \cos x \sin^3 x \, dx = \frac{u^4}{4} - \frac{u^2}{2} + C$

$\int \cos x \sin^3 x \, dx = \frac{[\cos x]^4}{4} - \frac{[\cos x]^2}{2} + C$

$\therefore \int \cos x \sin^3 x \, dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C$

$\int \cos x \sin^3 x \, dx = \frac{\cos^2 x}{2} \left[\frac{\cos^2 x}{2} - 1 \right] + C$

$= x - \frac{\sin 2x}{4} \frac{-24x + 18x - 5x}{16} + \frac{6\sin 4x - 3\sin 4x}{64}$

$+ \frac{\sin^3(2x)}{48} + c$

$\therefore \int \sin^6 x \, dx = x - \frac{\sin 2x}{4} - \frac{11x}{16} + \frac{3\sin 4x}{64}$

$+ \frac{\sin^3(2x)}{48} + c$

2 $\int \cos^4 x \sin^3 x \, dx$

Since n is odd, $u = \sin x$

$\frac{du}{dx} = \cos x \Rightarrow dx = \frac{du}{\cos x}$

And $\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\therefore \int \cos^4 x \sin^3 x \, dx$

$= \int [\cos^2 x][\cos^2 x] \, u^3 \cdot \frac{du}{\cos x}$

$= \int \cos x [\cos^2 x] \, u^3 \cdot du$

$= \int \cos x [1 - \sin^2 x] \, u^3 \cdot du$

but $\cos^2 x = 1 - \sin^2 x$. Hence, $\cos x = \sqrt{1 - \sin^2 x}$

$\cos x = [1 - \sin^2 x]^{1/2}$

$\therefore \int \cos x [1 - \sin^2 x] \, u^3 \cdot du$

$= \int [1 - \sin^2 x]^{1/2} [1 - \sin^2 x] \quad u^3 \cdot du$

$= \int [1 - \sin^2 x]^{3/2} \, u^3 \cdot du$

$= \int [1 - u^2]^{3/2} \, u^3 \cdot du$

$= \int [1 - u^2]^{3/2} [u^2]^{3/2} \cdot du$

$= \int [u^2 - u^4]^{3/2} \, du$

Let $P = u^2 - u^4$