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1) $\int \sin^6 2x\,dx$
2) $\int \cos^4 2x \sin^3 2x$
3) $\int \cos 2x \sin^3 2x$

1) $\int \sin^6 2x\,dx$

$\sin^6 2x = (\sin^2 2x)^2 (\sin^2 2x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$

$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$

$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$

$= \frac{1}{16}[3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x]$

$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \frac{1}{2} \cdot 2\cos 4x \cos 2x]$

$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)]$

$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)]$

$= \frac{1}{32}[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x]$

$= \frac{1}{32}[10 - 15\cos 2x + 6\cos 4x - \cos 6x]$

So, $\frac{1}{32}\int[10-15\cos 2x+6\cos 4x-\cos 6x]\,dx$

$= \frac{1}{32}\left[10x-\frac{15\sin 2x}{2}+\frac{6\sin 4x}{4}-\frac{\sin 6x}{6}\right]+C$

$\frac{1}{32}\left[10x-\frac{15\sin 2x}{2}+\frac{3\sin 4x}{2}-\frac{\sin 6x}{6}\right]+C$

$\frac{1}{192}\left[60x-45\sin 2x+9\sin 4x-\sin 6x\right]+C$

2) $\int \cos^4 x \sin^3 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x\,dx = -\frac{du}{\sin x}$

recall that $\sin^2(x)+\cos^2(x)=1$

$\sin^2(x) = 1-\cos^2(x)$

$= \int u^4(1-\cos^2(x))\,du$

~~$= \int u^4(1-u^2)\,du$~~ $\int(1-u^2)u^4\,du$

~~$= \int(u^4-u^6)\,du$~~ $\int(u^6-u^4)\,du$

~~$= u$~~ $= \frac{u^7}{7}-\frac{u^5}{5}$

Substitute $u=\cos x$

$= \frac{\cos^7 x}{7}-\frac{\cos^5 x}{5}$

$\cos^4 x \sin^3 x\,dx = \frac{\cos^7 x}{7}-\frac{\cos^5 x}{5}$

3) $\int \cos x \sin^3 x\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x \quad dx = -\frac{du}{\sin x}$

recall $\sin^2 x = 1 - \cos^2 x$

$$\int \cos x (1 - \cos^2 x)$$

$$\int -\cos x - 4\cos^3(x) \sin x$$

$$\int u - 4u^3 \cdot \sin x \, dx$$

$$\int (4u^3 - u) \, du$$

$$\frac{u^2}{2} - u^4$$

Substitute $u = \cos x$

$$\frac{\cos^2 x}{2} - \cos^4 x + c$$

$$\therefore \int \cos x \sin^3 x \, dx = \frac{\cos^2 x}{2} - \cos^4 x + c$$