Assignment (19/mhs01/016)
ADAM ELIAWIOCHE ADAM

① $\int \sin^6 x\,dx = \int \sin^6 x\,dx$
$= \int \sin x \cdot (\sin^2 x)^4\,dx$
since $\cos^2 x + \sin^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$
$= \int \sin x \cdot (1-\cos^2 x)^4\,dx$
$= \int \sin x \cdot [1 - 4\cos^2 x + 6\cos^4 x - 4\cos^6 x + \cos^8 x]\,dx$
$= \int \sin x - 4\sin x \cos^2 x + 6\sin x \cos^4 x - 4\sin x \cos^6 x + \sin x \cos^8 x$
$= -\cos x + \frac{4\cos^3 x}{3} - \frac{6\cos^5 x}{5} + \frac{4\cos^7 x}{7} - \frac{\cos^9 x}{9} + C$
$\therefore \int \sin^6 x = -\cos x + \frac{4\cos^3 x}{3} - \frac{6\cos^5 x}{5} + \frac{4\cos^7 x}{7} - \frac{\cos^9 x}{9} + C$

② $\int \cos^4 x \sin^3 x\,dx$
let $u = \cos x$
$\int \sin^3 x\, u^4\,dx$
but $\frac{du}{dx} = -\sin x$
$dx = -du/\sin x$
$= \int \sin x (\sin^2 x) u^4\,dx$
$= \int \sin x (\sin^2 x) u^4 \frac{-du}{\sin x}$
but $\cos^2 x + \sin^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$
$= -\int \sin x (1-\cos^2 x) u^4 \frac{du}{\sin x}$
$= -\int (1-\cos^2 x) u^4\,du$
but $u = \cos x$
$= -\int (1-u^2) u^4\,du$
$= \int ((-1+u^2) u^4\,du$
$= \int u^6 - u^4\,du$
$= \left[\frac{u^7}{7} - \frac{u^5}{5}\right] + C$
but $u = \cos x$
$\int \cos^4 x \sin^3 x\,dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

③ $\int \cos x \sin^3 x\,dx$
let $u = \sin x$
$\frac{du}{dx} = \cos x$
$du = \cos x\,dx$
$= \int u^3\,du$
$= \left[\frac{u^4}{4}\right] + C$
but $u = \sin x$
$\int \cos x \sin^3 x = \frac{\sin^4 x}{4} + C$