WOKE JEDIDIAH CHIMENEM 19/MHS01/438

(i) $\cos^4 x \sin^3 x$

(ii) $\cos x \sin^3 x$

Solution

i-)

$$\int \sin^6 x \, dx$$

$$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$$

$$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{8}(1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)$$

$$= \frac{1}{16}(2 - 4\cos 2x + 1 + \cos 4x)(1-\cos 2x)$$

$$= \frac{1}{16}(3 - 4\cos 2x + \cos 4x)(1-\cos 2x)$$

$$= \frac{1}{16}[3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x]$$

$$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \tfrac{1}{2} \cdot 2\cos 4x \cos 2x]$$

$$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2(1+\cos 4x) - \tfrac{1}{2}(\cos 6x + \cos 2x)]$$

$$= \frac{1}{16}[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \tfrac{1}{2}(\cos 6x + \cos 2x)]$$

$$= \frac{1}{32}[6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x]$$

$$= \frac{1}{32}[10 - 15\cos 2x + 6\cos 4x - \cos 6x$$

So,

$$I = \tfrac{1}{32}\int [10 - 15\cos 2x + 6\cos 4x - \cos 6x]\,dx$$

$$I = \tfrac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + c$$

$$I = \tfrac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + c$$

$$I = \tfrac{1}{192}[60x - 45\sin 2x + 9\sin 4x - \sin 6x] + c$$

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11.) $\int \cos^4 x \sin^3 x \, dx$

Recall that $\sin^2 x = 1 - \cos^2 x$

$= \int \cos^4 x (1 - \cos^2 x) \cdot \sin x \, dx$

let $u = \cos x$, $du/dx = -\sin x$

$\therefore dx = \frac{du}{-\sin x}$

$= \int \cos^4 x (1 - \cos^2 x) \cdot \sin x \cdot \frac{du}{-\sin x}$

$= \int \cos^4 x (\cos^2 x - 1) \, du$

Recall that $u = \cos x$

$= \int u^4 (u^2 - 1) \, du$

$= \int (u^6 - u^4) \, du$

Using power rule

$u^6 = \frac{u^{6+1}}{6+1}$

$u^4 = \frac{u^{4+1}}{4+1}$

$= \frac{u^7}{7} - \frac{u^5}{5} + c$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

OR $\frac{5\cos^7 x - 7\cos^5 x}{35} + c$

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iii.) $\int \cos x \sin^3 x \, dx$

$= \sin x, \ d/dx = \cos x$

let $u = \sin x, \ du/dx = \cos x$

$\therefore du = \cos x \, dx$

$\therefore \int \sin^3 x \cos x \, dx = \int u^3 du$ ($\int du \cdot u^3$)

Using reverse power rule:

$$\int u^3 du = \frac{u^{3+1}}{3+1} + c$$

$$\int u^3 du = \frac{u^4}{4} + c$$

Recall that $u = \sin x$

$$\int \sin^3 x \underbrace{\cos x \, dx}_{\int \cos x \sin^3 x \, dx} = \frac{\sin^4 x}{4} + c$$

$$= \frac{\sin^4 x}{4} + c$$