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100 LEVEL

MATRIC NUMBER: 19/MHS 01/216

MAT 104 ASSIGNMENT

Integrate the following functions.

$$\sin^6 x$$

Solution

Using the formula:

$$\int \sin(x)^n dx = -\frac{1}{n} \sin(x)^{n-1} \cos(x) + \frac{n-1}{n} \int \sin(x)^{n-2} dx$$

$$\int \sin^6 x dx = -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \int \sin^4 x dx$$

$$\int \sin^4 x dx = -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left(-\frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \int \sin^2 x dx \right)$$

$$\text{Using } \sin^2 x = 1 - \cos 2x$$

$$\int \sin^6 x dx = -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left(-\frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \int (1 - \cos 2x) dx \right)$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left(-\frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \cdot \frac{1}{2} \int (1 - \cos 2x) dx \right)$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left[-\frac{1}{4} \sin^3 x \cos x + \frac{3}{8} \int (1 - \cos 2x) dx \right]$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left(-\frac{1}{4} \sin^3 x \cos x + \frac{3}{8} \left(x - \frac{\sin 2x}{2} \right) \right)$$

$$= -\frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \left[-\frac{1}{4} \sin^3 x \cos x + \frac{3}{8} x - \frac{3 \sin 2x}{16} \right]$$

$$= -\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x + \frac{5x}{16} - \frac{5 \sin 2x}{32} + C$$

$$\therefore \int \sin^6 x dx = -\frac{\sin^5 x \cos x}{6} - \frac{5 \sin^3 x \cos x}{24} + \frac{5}{16} x - \frac{5 \sin 2x}{32} + C$$

$$2 \int \cos^4 x \sin^3 x \, dx$$

Solution :

Since m is odd, $u = \cos x$
 $\frac{du}{dx} = -\sin x \Rightarrow dx = -\frac{du}{\sin x}$

$$\begin{aligned}\int \cos^4 x \sin^3 x \, dx &= \int \sin x \cdot \sin^2 x \times u^4 \cdot -\frac{du}{\sin x} \\&= \int (1 - \cos^2 x) \cdot -u^4 du \\&= \int (1 - u^2) \cdot -u^4 du \\&= \int (u^6 - u^4) du \\&= \frac{u^7}{7} - \frac{u^5}{5} + C\end{aligned}$$

$$\therefore \int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

$$3 \int \cos x \sin^3 x \, dx$$

Solution

Since m is odd, $u = \cos x$
 $\frac{du}{dx} = -\sin x \Rightarrow dx = -\frac{du}{\sin x}$

$$\begin{aligned}\int \cos x \sin^3 x \, dx &= \int \sin x \cdot \sin^2 x \times u \cdot -\frac{du}{\sin x} \\&= \int (1 - \cos^2 x) \cdot -u du \\&= \int (1 - u^2) \cdot -u du \\&= \int (u^3 - u) du \\&= \frac{u^4}{4} - \frac{u^2}{2} + C\end{aligned}$$

$$\int \cos x \sin^3 x \, dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C$$