NAME: EKANEM, ENOABASI NKEREUWEM

MATRIC NO: 19/MHS01/146

DEPARTMENT: MBBS

COLLEGE: MHS

COURSE: MAT 104

ANSWER TO ASSIGNMENT

1. $\sin^6 x$

Solution

$$\int \sin^6(x)\,dx$$

Apply reduction formula

$$\int \sin^n(x)\,dx = \frac{n-1}{n}\int \sin^{n-2}(x)\,dx - \frac{\cos(x)\sin^{n-1}(x)}{n}$$

with $n=6$

$$-\frac{\cos(x)\sin^5(x)}{6} + \frac{5}{6}\int \sin^4(x)\,dx$$

or choose an alternative (product to sum formula)

$$\int \sin^4(x)\,dx$$

Apply the last reduction formula again with $n=4$

$$-\frac{\cos(x)\sin^3(x)}{4} + \frac{3}{4}\int \sin^2(x)\,dx$$

$$\int \sin^2(x)\,dx$$

Apply the reduction formula again with $n=2$

$$-\frac{\cos(x)\sin(x)}{2} + \frac{1}{2}\int 1\,dx$$

or choose an alternative (product to sum formula)

$$\int 1\,dx$$

Apply constant rule $= x$

Add with solved integrals

$$\frac{-\cos(x)\sin(x)}{2} + \frac{1}{2}\int 1\,dx$$

$$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$$

Add in solved integrals

$$\frac{-\cos(x)\sin^3(x)}{4} + \frac{3}{4}\int \sin^2(x)\,dx$$

$$-\frac{\cos(x)\sin^3(x)}{4} - \frac{3\cos(x)\sin(x)}{8} + \frac{3x}{8}$$

Add in solved integrals

$$\frac{-\cos(x)\sin^5(x)}{6} + \frac{5}{6}\int \sin^4(x)\,dx$$

$$-\frac{\cos(x)\sin^5(x)}{6} - \frac{5\cos(x)\sin^3(x)}{24} - \frac{5\cos(x)\sin(x)}{16} + \frac{5x}{16}$$

$$= \frac{-\cos(x)\sin^5(x)}{6} - \frac{5\cos(x)\sin^3(x)}{24} - \frac{5\cos(x)\sin(x)}{16} + \frac{5x}{16} + C$$

$$= \frac{-\sin(6x) - 9\sin(4x) + 45\sin(2x) - 60x}{192} + C$$

2. Integration of given function $\cos^4 x \sin^3 x$

Solution

$$\int \cos^4(x)\sin^3(x)\,dx$$

Substitution, use $\sin^2(x) = 1 - \cos^2(x)$

$$\int -\cos^4(x)(\cos^2(x) - 1)\cdot \sin(x)\,dx$$

Substitute $u = \cos(x) \longrightarrow du/dx = -\sin(x)$

$$dx = \frac{-1}{\sin(x)}\,du$$

$$= \int u^4(u^2 - 1)\,du$$

choose an alternative

2.)

Expand

$= \int (u^6 - u^4)\, du$

Apply linearity

$= \int u^6 du - \int u^4 du$

$\int u^6 du$

Apply power rule

$\int u^n du = \frac{u^{n+1}}{n+1}$ with $n = 6$

$= \frac{u^7}{7}$

$\int u^4 du$

Apply power rule with $n = 4$

$\therefore \frac{u^5}{5}$

Add in solved integrals

$\int u^6 du - \int u^4 du$

$= \frac{u^7}{7} - \frac{u^5}{5}$

Undo substitution $u = \cos(x)$

$= \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$

$\int \cos^4(x) \sin^3(x) dx$

$= \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5} + C$

$= \frac{5\cos^7(x) - 7\cos^5(x)}{35} + C$

3. Integration of given function $\cos x \sin^3 x$

Solution

$$\int \cos(x) \sin^3(x)\, dx$$

Substitute $u = \sin(x) \longrightarrow du/dx = \cos(x)$

$$\longrightarrow dx = \frac{1}{\cos(x)} du \quad : \quad = \int u^3 du$$

Apply power rule

$$\int u^n du = \frac{u^{n+1}}{n+1} \quad \text{with } n=3$$

$$= \frac{u^4}{4}$$

Undo substitution $u = \sin(x)$

$$= \frac{\sin^4(x)}{4}$$

$$\therefore \int \cos(x) \sin^3(x)\, dx$$

$$= \frac{\sin^4(x)}{4} + c$$