ASSIGNMENT

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$\int \sin^6 x \, dx$

solution

$$\int \sin^6 x \, dx = (\sin^2 x)^2 (\sin^2 x)$$

$$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \cos^2 2x\right)\left(1-\cos 2x\right)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)\left(1-\cos 2x\right)$$

$$= \frac{1}{16}\left(2 - 4\cos 2x + 1 + \cos 4x\right)\left(1-\cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 4\cos 2x + \cos 4x\right)\left(1-\cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cdot \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{1}{2}2\cos 4x\cos 2x\right)$$

$$= \frac{1}{32}\left(6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x\right)$$

$$= \frac{1}{32}\left(10 - 15\cos 2x + 6\cos 4x - \cos 6x\right)$$

let $\sin^6 x = R$

$$\therefore R = \frac{1}{32}\int \left(10 - 15\cos 2x + 6\cos 4x - \cos 6x\right) dx$$

$$R = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$$

② $\int \cos^4 x \sin^3 x \, dx$

Solution:

$$\int \cos^4 x \sin^3 x \, dx$$

Since $m = \text{odd}$

$u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow dx = -\frac{du}{\sin x}$

And

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$\int \cos^4 x \sin^3 x \, dx = \int \cancel{\sin x} \cdot \sin^2 x \cdot u^4 \cdot -\frac{du}{\cancel{\sin x}}$$

$$= -\int \sin^2 x \cdot u^4 \cancel{x} \, du$$

$$= -\int (1 - \cos^2 x) \cdot u^4 \, du$$

$$= -\int (1 - u^2) \cdot u^4 \, du$$

$$= -\int (u^4 - u^6) \cdot du$$

$$= \int (u^6 - u^4) \, du$$

$$= \left[\frac{u^7}{7} - \frac{u^5}{5}\right] + C$$

$$\int \cos^4 x \sin^3 x \, dx = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

③ $\int \cos x \sin^3 x \, dx$

Solution:

$$\int \cos x \sin^3 x \, dx$$

Since $n = \text{odd}$

$u = \sin x$

$du/dx = \cos x \Rightarrow du = \cos x \, dx$

$$= \int u^3 \times du$$

$$= \left(\frac{u^4}{4} + c\right)$$

$$\therefore \int \cos x \sin^3 x \, dx = \frac{(\sin x)^4}{4} + c$$

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