UNUANE - HENRY OSEREMHEN C.

19/MHS01/424 MAT 104 Assignment MBBS

1 $\int \sin^6 x$

$$\int \sin^6 x = \int \sin^4 x \cdot \sin^2 x$$

$$\int \sin^6 x = \int (\sin^2 x)^2 \cdot \sin^2 x$$

But $\sin^2 x = \frac{1 - \cos 2x}{2}$

$$\therefore \int \sin^6 x = \int \left(\frac{1 - \cos 2x}{2}\right)^2 \cdot \left(\frac{1 - \cos 2x}{2}\right)$$

$$= \int \frac{(1 - \cos 2x)^2}{4} \cdot \frac{(1 - \cos 2x)}{2}$$

$$= \int \frac{(1 - \cos 2x)^3}{8}$$

$$= \frac{1}{8} \int (1 - \cos 2x)^3$$

$$= \frac{1}{8} \int (1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - \cos 2x - 2\cos 2x + 2\cos^2 2x + \cos^2 2x - \cos^3 2x$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x$$

$$\int \sin^6 x = \frac{1}{8} \int 1 - 3\cos 2x + 3\left(\frac{1 - \cos 2(2x)}{2}\right) - (\cos^2 2x \cdot \cos 2x)$$

$$= \frac{1}{8} \int 1 - 3\cos 2x + 3\left(\frac{1 - \cos 4x}{2}\right) - (1 - \sin^2 2x)(\cos 2x)$$

$$= \frac{1}{8} \int 1 - 3\cos 2x + \frac{3}{2} - \frac{3\cos 4x}{2} - \cos 2x + \cos 2x \sin^2 2x$$

$$= \frac{1}{8} \int \frac{5}{2} - 4\cos 2x - \frac{3\cos 4x}{2} + \underline{\cos 2x \sin^2 2x}$$

$$\int \sin^6 x = \frac{1}{8}\int \frac{5}{2} - 4\cos 2x - \frac{3\cos 4x}{2} + \int \cos 2x \sin^2 2x \, dx$$

$$\int \sin^6 x = \frac{1}{8}\left[\frac{5x}{2} - \frac{4\sin 2x}{2} - \frac{3\sin 4x}{4\times 2}\right] +$$

Let $u = \sin 2x$

$$\frac{du}{dx} = \frac{\cos 2x}{2}$$

$$\frac{dx}{du} = \frac{2}{\cos 2x}$$

$$\int \sin^6 x = \frac{1}{8}\left[\frac{5x}{2} - 2\sin 2x - \frac{3\sin 4x}{8} + \right.$$

$$\therefore dx = \frac{2\,du}{\cos 2x}$$

$$= \int \cancel{\cos 2x} \cdot u^2 \cdot \frac{2du}{\cancel{\cos 2x}}$$

$$= \int 2u^2 \, du$$

$$= \frac{2u^3}{3} + c$$

But $u = \sin 2x$

$$= \frac{2(\sin 2x)^3}{3} + c$$

$$\therefore \int \sin^6 x = \frac{1}{8}\left[\frac{5x}{2} - 2\sin 2x - \frac{3\sin 4x}{8} + \frac{2\sin^3 2x}{3}\right] + c$$

$$\int \sin^6 x = \frac{5x}{16} - \frac{\sin 2x}{4} - \frac{3\sin 4x}{64} + \frac{\sin^3 2x}{12} + c$$

b $\int \cos^4 x \sin^3 x$

~~Let $u = \cos 4x$~~

Let $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$\frac{dx}{du} = \frac{-1}{\sin x} \qquad \therefore dx = \frac{-du}{\sin x}$$

$$= \int u^4 \cdot \sin^3 x \, dx$$

$$= \int u^4 \cdot \cancel{\sin x} \cdot \sin^2 x \cdot \frac{-du}{\cancel{\sin x}}$$

$$= -\int u^4 \sin^2 x \, du$$

$$= -\int u^4 (1 - \cos^2 x) \, du$$

$$= -\int u^4 (1 - u^2) \, du$$

$$= -\int u^4 - u^6 \, du$$

$$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + c$$

$$= \frac{-u^5}{5} + \frac{u^7}{7} + c$$

But $u = \cos x$

$$\int \cos^4 x \sin^3 x = -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + c$$

c $\int \cos x \sin^3 x \, dx$

Let $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x \, dx$$

In $\int \cos x \sin^3 x \, dx = \int \cos x \, u^3 dx$

But $du = \cos x \, dx$

$$\therefore = \int u^3 du$$

$$= \frac{u^4}{4} + c$$

$$= \frac{\sin^4 x}{4} + c$$