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MATRIC NO: 19/MHS01/107
DEPARTMENT: MEDICINE AND SURGERY
COLLEGE: MEDICINE AND HEALTH SCIENCES
COURSE: MAT 104.

ASSIGNMENT.

1 $\int \sin^6 x\,dx$

Solution

$$\int \sin^6 x\,dx = \int \sin^4 \cdot \sin^2 x$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin^4 x = \left[\frac{1-\cos 2x}{2}\right]^2$$

$$\sin^4 x = \frac{1}{4}\left[(1-\cos 2x)^2\right]$$

$$\sin^4 x = \frac{1}{4}\left[1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right]$$

$$\int \sin^6 x\,dx = \frac{1}{4}\int\left[1-2\cos 2x+\frac{1+\cos 4x}{2}\right]\cdot\left[\frac{1-\cos 2x}{2}\right]dx$$

$$\int \sin^6 x\,dx = \frac{1}{8}\int\left(1-2\cos 2x+\frac{1+\cos 4x}{2}\right](1-\cos 2x)\,dx$$

$$\int \sin^6 x\,dx = \frac{1}{16}\int(2-4\cos 2x+1+\cos 4x)(1-\cos 2x)\,dx$$

$$\int \sin^6 x\,dx = \frac{1}{16}\int(3-4\cos 2x+\cos 4x)(1-\cos 2x)\,dx$$

$$\int \sin^6 x\,dx = \frac{1}{16}\int(3-4\cos 2x+\cos 4x-3\cos 2x+4\cos^2 4x-\cos 4x\cos 2x)\,dx$$

$$\int \sin^6 x\,dx = \frac{1}{16}\int(3-4\cos 2x+\cos 4x-3\cos 2x+2\times 2\cos^2 2x-\cos 4x\cos 2x)\,dx$$

$$\cos^2 2x = \frac{1+\cos 4x}{2}$$

$\cos 4x \cos 2x = \frac{1}{2}\cos(4x+2x) + \cos(4x-2x)$

$\cos 4x \cos 2x = \frac{1}{2}(\cos 6x + \cos 2x)$

$\int \sin^6 x\,dx = \frac{1}{16}\int\left(3 - 7\cos 2x + \cos 4x + 2 \times \cancel{2}\left(\frac{1+\cos 4x}{\cancel{2}}\right] - \frac{1}{2}(\cos 6x + \cos 2x)\right\}dx$

$\int \sin^6 x\,dx = \frac{1}{16}\int\left\{3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right\}dx$

$\int \sin^6 x\,dx = \frac{1}{32}\int(6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x + \cos 2x)\,dx$

$\int \sin^6 x\,dx = \frac{1}{32}\int(10 - 15\cos 2x + 6\cos 4x - \cos 6x)\,dx$

$\int \sin^6 x\,dx = \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + C$ //

where c is the constant of integration.

2 $\int \cos^4 x \sin^3 x\,dx$.

Solution

~~Since~~ Let $u = \cos x$ $\quad \frac{du}{dx} = -\sin x \quad dx = \frac{-du}{\sin x}$

Substituting the values of u and dx into $\int \cos^4 x \sin^3 x\,dx$;

$\int \cos^4 x \sin^3 x\,dx = \int u^4 \cdot \sin^2 x \cdot \cancel{\sin x} \cdot \frac{-du}{\cancel{\sin x}}$

$\int \cos^4 x \sin^3 x\,dx = -\int u^4 \sin^2 x\,du$

$\sin^2 x = 1 - \cos^2 x$

$\int \cos^4 x \sin^3 x\,dx = -\int u^4(1 - \cos^2 x)\,du$

$\int \cos^4 x \sin^3 x\,dx = -\int u^4(1 - u^2)\,du$

$\int \cos^4 x \sin^3 x\,dx = \int(u^6 - u^4)\,du$

$$\int \cos^4 \sin^3 x\,dx = \frac{u^7}{7} - \frac{u^5}{5} + c$$

$$\int \cos^4 \sin^3 x\,dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$ // where c is the constant of integration.

3 $\int \cos x \sin^3 x\,dx$

<u>Solution</u>

Let $u = \cos x$ $\quad \frac{du}{dx} = -\sin x \quad dx = \frac{-du}{\sin x}$

$$\int \cos x \sin^3 x\,dx = \int u \cdot \sin^2 x \cdot \cancel{\sin x} \cdot \frac{-du}{\cancel{\sin x}}$$

$$\int \cos x \sin^3 x\,dx = -\int u \cdot \sin^2 x\,du$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\int \cos x \sin^3 x\,dx = -\int u(1 - \cos^2 x)\,du$$

$$\int \cos x \sin^3 x\,dx = -\int u(1 - u^2)\,du$$

$$\int \cos x \sin^3 x\,dx = \int (u^3 - u)\,du$$

$$\int \cos x \sin^3 x\,dx = \left[\frac{u^4}{4} - \frac{u^2}{2}\right] + c$$

$$\int \cos x \sin^3 x\,dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + c$$ // where c is the constant of integration.