1 $\sin^6 x$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$\int \sin^6 x = \int \left(\left(\frac{1-\cos 2x}{2} \right)^2 \left(\frac{1-\cos 2x}{2} \right) \right.$

$\sin^6 x = \frac{1}{8} \int (1-\cos 2x)^2 (1-\cos 2x)\, dx$

$\sin^6 x = \frac{1}{8} \int (1 - 2\cos 2x + \cos^2 2x)(1-\cos 2x)\, dx$

$\sin^6 x = \frac{1}{8} \int \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)(1-\cos 2x)\, dx$

$\sin^6 x = \frac{1}{16} \int (2 - 4\cos 2x + 1 + \cos 4x)(1-\cos 2x)\, dx$

$\sin^6 x = \frac{1}{16} \int (3 - 4\cos 2x + \cos 4x)(1-\cos 2x)\, dx$

$\sin^6 x = \frac{1}{16} \int (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos 2x$

$- \cos 4x \cos 2x)\, dx$

$\sin^6 x = \frac{1}{16} \int (3 - 7\cos 2x + \cos 4x + 2(2\cos^2 2x) - \frac{3}{}$

$- \frac{1}{2} 2\cos 4x \cos 2x)\, dx$

$\sin^6 x = \frac{1}{16} \int (3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x$

$- \frac{1}{2} (\cos 6x + \cos 2x)\, dx$

$\sin^6 x = \frac{1}{32} \int (6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4 - \cos 6x$

$- \cos 2x)\, dx$

$$\sin^6 x = \frac{1}{32}\int\left(10 - 15\cos 2x + 6\cos 4x - \cos 6x\right)dx$$

$$\sin^6 x = \frac{1}{32}\left(10x - \frac{15\sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$$

(2) $\cos^4 x \sin^3 x$

$U = \cos x$

$\frac{du}{dx} = -\sin x \qquad dx = \frac{-du}{\sin x}$

$$\int \cos^4 x \sin^3 x \, dx = \int U^4 \cdot \cancel{\sin x} \cdot \sin^2 x \cdot \frac{-du}{\sin x}$$

$$= \int -U^4 \cdot (\sin^2 x) \, du$$

$$= \int -U^4 \cdot (1 - \cos^2 x] \, du$$

$$= \int -U^4 \cdot (1 - U^2) \, du$$

$$= \int (-U^4 + U^6) \, du$$

$$= \left[\frac{U^7}{7} - \frac{U^5}{5} \right] + C$$

$$\int \cos^4 x \sin^3 x \, dx = \left[\frac{\cos^7 x}{7} - \frac{\cos^5 x}{7} \right] + C$$

3 $\int \cos x \sin^3 x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int \cos x \sin^3 x \, dx = \int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$

$\int \cos x \sin^3 x \, dx = \int u^3 \cdot du$

$\int \cos x \sin^3 x \, dx = \frac{u^4}{4} + C$

$\int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + C$