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141m H501/342

$$\begin{aligned}
 \sin^6 x &= (\sin^2 x)(\sin^2 x)^2 \\
 &= \left(\frac{1-\cos 2x}{2}\right) \left(\frac{1-\cos 2x}{2}\right)^2 \\
 &= \frac{1}{8} (1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x) \\
 &= \frac{1}{8} (1 - 2\cos 2x + \frac{1+\cos 4x}{2})(1 - \cos 2x) \\
 &= \frac{1}{16} (2 - 4\cos 2x + \cos 4x)(1 - \cos 2x) \\
 &= \frac{1}{16} (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x) \\
 &= \frac{1}{16} (3 - 7\cos 2x + \cos 4x - 2(\cos 2x) - 4\cos 4x \cos 2x) \\
 &= \frac{1}{16} (3 - 7\cos 2x + \cos 4x + 2(1 - \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)) \\
 &= \frac{1}{16} (3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)) \\
 &= \frac{1}{32} (6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x) \\
 &= \frac{1}{32} (10 - 15\cos 2x + 6\cos 4x - \cos 6x)
 \end{aligned}$$

Let  $\sin^2 x = R$

$$R = \frac{1}{32} \int (10 - 15\cos 2x + 6\cos 4x - \cos 6x) dx$$

$$R = \frac{1}{32} \left( 10x - 15 \frac{\sin 2x}{2} + 6 \frac{\cos 4x}{4} - \frac{\cos 6x}{6} \right) + C$$

$$\int \sin^6 x dx = \frac{\cos x}{32} - \frac{15 \sin 2x}{64} + \frac{6 \cos 4x}{128} - \frac{\cos 6x}{192} + C$$

$$2) \cos^4 x \sin^3 x$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$dx = -\frac{du}{\sin x}$$

$$\int \cos^4 x \sin^2 x \cdot \sin x \cdot -\frac{du}{\sin x}$$

$$-\int \cos^4 x \sin^2 x \, du$$

$$-\int u^4 (1 - \cos^2 x) \, du$$

$$-\int u^4 (1 - u^2) \, du$$

$$-\int u^4 - u^6 \, du$$

$$-\frac{u^5}{5} + \frac{u^7}{7} + C$$

$$\frac{u^7}{7} - \frac{u^5}{5} + C$$

$$\frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

$$3) \cos x \sin^3 x$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$dx = \frac{du}{\cos x}$$

$$\int \cos x \sin^2 x \cdot \sin x \cdot \frac{du}{\cos x}$$

$$\int \sin x \sin^2 x du \quad u^3$$

$$\int \sin x \cos x = \frac{u^4}{4} + C$$

$$\frac{(\sin x)^4}{4} + C$$