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course: Math 104
Dept: MBBS.

Integrate the following.

a. $\int \sin^6 x\,dx$

$= \sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

Recall that: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$= \left(\frac{1 - \cos 2x}{2}\right)^2 \left(\frac{1 - \cos 2x}{2}\right)$

$= \frac{1}{2}(1 - \cos 2x)^2 \cdot \frac{1}{2}(1 - \cos 2x)$

$= \frac{1}{8}\int (1 - \cos 2x)^2 \cdot (1 - \cos 2x)$

$\rightarrow \frac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x) \cdot (1 - \cos 2x)$

$\frac{1}{8}\int \left(1 \cdot 2\cos 2x + \left(\frac{1 + \cos 2(2x)}{2}\right) \cdot (1 - \cos 2x)\right)$

$\frac{1}{8}\int \left(1 \cdot 2\cos 2x + \frac{1 + \cos 4x}{2}\right) \cdot (1 - \cos 2x)$

$\frac{1}{16}\int (2 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$

$\frac{1}{16}\int (3 - 4\cos 2x + \cos 4x)(1 - \cos 2x)$

$\frac{1}{16}\int (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$

$\frac{1}{16}\int (3 - 7\cos 2x + \cos 4x + (2 \cdot 2\cos^2 2x) - \frac{1}{2} \cdot 2\cos 4x \cos 2x)$

$\frac{1}{16}\int 3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)$

$\frac{1}{32}\int (6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x)$

$\rightarrow \frac{1}{32}\int (10 - 14\cos 2x + 2\cos 4x + 4\cos 4x - \cos 6x - \cos 2x)$

$\frac{1}{32}\int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)\,dx$

$\frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + c$

$\frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + c$

$= \int \sin^6 x\,dx$

$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{3\sin 4x}{2} - \frac{\sin 6x}{6}\right] + C$

2. $\int \cos^4 x \sin^3 x\,dx$

$\rightarrow \int \cos^4 x \cdot \sin^2 x \cdot \sin x\,dx$

Recall that $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\int \cos^4 x \cdot \frac{1}{2}(1 - \cos 2x) \cdot \sin x\,dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x = du = -\sin x\,dx$

$-du = \sin x\,dx$

$\rightarrow 2u^2 - 1 = \cos 2x$

$\frac{-1}{2}\int u^4 \cdot (1 \cdot (2u^2 - 1))\,du$

$-\frac{1}{2}\int u^4 \cdot (1 - 2u^2 + 1)\,du$

$-\frac{1}{2}\int u^4 \cdot (2 - 2u^2)\,du$

$-\frac{1}{\cancel{2}}\int u^4 \cdot \cancel{2}(1 - u^2)\,du$

$-\int u^4 (1 - u^2)\,du$

$-\int u^4 - u^6\,du$

$-\frac{u^5}{5} - \frac{u^7}{7} + c$

$-\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

$\rightarrow \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + C$

$\int \cos^4 x \sin^3 x\,dx = \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + c$

3. $\int \cos x \sin^3 x \, du$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\Rightarrow du = \cos x \, dx$

$\int u^3 . du$

$\rightarrow \frac{u^{3+1}}{3+1} + c$

$\rightarrow \frac{u^4}{4} + c$

$= \frac{\sin^4 x}{4} + c$

$\rightarrow \int \cos x \sin^3 x \, dx = \frac{\sin^4 x}{4} + c$