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19/MHS01/412
MBBS
MAT104 Assignment

1. $\sin^6 x$

$\sin^2 x = 1 - \cos^2 x$

$\int (1-\cos^2 x)^3 dx$

$\int (1-\cos^2 x)^2 (1-\cos^2 x)\, dx$

$\int (1 - 2\cos^2 x + \cos^4 x)(1-\cos^2 x)\, dx$

$\int (1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x)\, dx$

using $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$$\int 1 - 3\left(\frac{1}{2}(1+\cos 2x)\right) + 3\left(\frac{1}{4}(1+\cos 2x)^2\right) - \left(\frac{1}{8}(1+\cos 2x)^3\right) dx$$

$$\int 1 - \frac{3}{2}(1+\cos 2x) + \frac{3}{4}(1+\cos 2x)^2 - \frac{1}{8}(1+\cos 2x)^3\, dx$$

$$\int 1 - \frac{3}{2} - \frac{3}{2}\cos 2x + \frac{3}{4}(1 + 2\cos 2x + \cos^2 2x) - \frac{1}{8}(1 + 3\cos 2x + 3\cos^2 2x + \cos^3 2x)\, dx$$

$$\int 1 - \frac{3}{2} - \cancel{\frac{3}{2}\cos 2x} + \frac{3}{4} + \cancel{\frac{3\cos 2x}{2}} + \frac{3}{4}\cos^2 2x - \frac{1}{8} - \frac{3}{8}\cos 2x - \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos^3 2x\, dx$$

$$\int \frac{1}{8} + \frac{3\cos^2 2x}{4} - \frac{3}{8}\cos 2x - \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos^3 2x\, dx$$

$$\int \frac{1}{8} + \frac{3}{4}\frac{(1+\cos 4x)}{2} - \frac{3}{8}\cos 2x - \frac{3}{8}\frac{(1+\cos 4x)}{2} - \frac{1}{8}(\cos^2 2x \cdot \cos 2x)\, dx$$

$$\int \frac{1}{8} + \frac{3}{8} + \frac{3\cos 4x}{8} - \frac{3\cos 2x}{8} - \frac{3}{16} - \frac{3}{16}\cos 4x - \frac{1}{8}(\cos^2 2x \cdot \cos 2x)\, dx$$

$$\int \frac{5}{16} + \frac{3\cos 4x}{8} - \frac{3\cos 2x}{8} - \frac{3\cos 4x}{16} - \frac{1}{8}(\cos^2 2x \cdot \cos 2x)\, dx$$

$$\int \frac{5}{16} + \frac{3}{8}\cos 4x - \frac{3\cos 2x}{8} - \frac{3}{16}\cos 4x - \frac{1}{8}(1 - \sin^2 2x)(\cos 2x)\, dx$$

$$\int \frac{5}{16} + \frac{3\cos 4x}{8} - \frac{3\cos 2x}{8} - \frac{3}{16}\cos 4x - \frac{1}{8}\int (1-\sin^2 2x)(\cos 2x)\, dx$$

$$\int \frac{5}{16} + \frac{3}{8}\cos 4x - \frac{3}{8}\cos 2x - \frac{3}{16}\cos 4x$$

$$= \frac{5x}{16} + \frac{3\sin 4x}{32} - \frac{3}{16}\sin 2x - \frac{3}{64}\sin^4 x$$

Finding $\int (1-\sin^2 2x)(\cos 2x)$

let $y = \sin 2x$; $\frac{dy}{dx} = 2\cos 2x$

$\frac{1}{2} dy = \cos 2x \, dx$

$\frac{1}{2}\int 1 - y^2 \, dy$

$= \frac{1}{2}\left(y - \frac{y^3}{3}\right)$

$= \frac{y}{2} - \frac{y^3}{6}$

$= \frac{\sin 2x}{2} - \frac{\sin^3 2x}{6}$

Add everything

$= \frac{5x}{16} + \frac{3}{32}\sin 4x - \frac{3}{16}\sin 2x - \frac{3}{64}\sin 4x - \frac{1}{8}\left(\frac{\sin 2x}{2} - \frac{\sin^3 2x}{6}\right) + c$

$= \frac{5x}{16} + \frac{3}{32}\sin 4x - \frac{3\sin 2x}{16} - \frac{3}{64}\sin 4x - \frac{\sin 2x}{16} + \frac{\sin^3 2x}{48} + c$

$= \frac{60x + 18\sin 4x - 36\sin 2x - 9\sin 4x - 12\sin 2x + 4\sin^3 2x}{192} + c$

$\int \sin^6 x = \frac{60x + 9\sin 4x - 48\sin 2x + 4\sin^3 2x}{192} + c$

2) $\int \cos^4 x \sin^3 x \, dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x \quad dx = \frac{-du}{\sin x}$

$= \int u^4 \cdot \sin^3 x \cdot \frac{-du}{\sin x}$

$= -\int u^4 \cdot \sin^2 x \, du$

$\sin^2 x = 1 - \cos^2 x$

$= -\int u^4(1-u^2)\,du$

$= -\int (u^4 - u^6)\,du$

$= \int u^6 - u^4$

$= \frac{u^7}{7} - \frac{u^5}{5}$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$

$\int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3.) $\int \cos x \sin^3 x \, dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$\int u \cdot \sin^3 x \cdot \frac{-du}{\sin x}$

$= -\int u \sin^2 x \cdot du$

$= -\int u(1 - \cos^2 x) du$

$= -\int u(1 - u^2) du$

$= -\int u - u^3 du$

$= \int u^3 - u$

$= \frac{u^4}{4} - \frac{u^2}{2} + c$

$\int \cos x \sin^3 x \, dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + c$